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PURIFICATION OF WATER FOR BOILER FEED PURPOSES.*

BY

FREDERICK G. STRAUB, Met.E.,

University of Illinois.

The problem of purification of water to be used for feed purposes in steam boilers did not become one of major importance until about twenty years ago. This was due to the fact that steam generation was in general just incidental to the operation of an industrial plant or utility station. Gradually as the efficiency of the other parts of the industry increased it was noted that marked savings were possible in the steam generation part of the industry. The central station being primarily interested in steam and power generation was naturally the first to explore the possibilities of increased efficiency in the generation and utilization of steam.

In 1917 George F. Gebhardt [1] describing what he termed a modern steam turbine plant cited Northwest Station of the Commonwealth Edison Co., Chicago, Illinois, as such. This plant had a turbine-generator of 30,000 Kilowatt rated capacity. The steam for the turbine was generated in five boilers at 250 lbs. per sq. in. pressure and a final temperature of 625° F. When operating at full capacity 400,000 lbs. of steam per hour could be produced or 80,000 lbs. of steam per boiler.

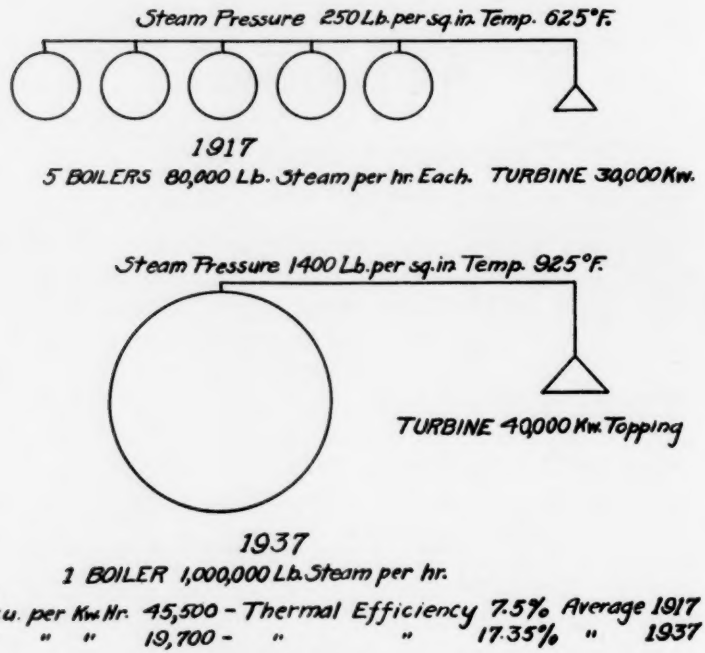
* Presented at a meeting held Thursday November 3, 1938.

[1] "Steam Power Plant Engineering," Gebhardt, 1917. Pg. 16.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

Electrical world in Jan. 15, 1938, in summing the economy and efficiency of the central station industry in the United States, stated that the average B.T.U. per KW. hour in 1917 was 45,500 and the thermal efficiency was 7.51 per cent. while in 1937 the B.T.U. per KW. hour had been reduced to 19,700 B.T.U. and the thermal efficiency increased to 17.35 per cent. These great changes in efficiency have been ac-

FIG. 1.


Steam Pressure 250 Lb. per sq. in. Temp. 625° F.
1917
5 BOILERS 80,000 Lb. Steam per hr. Each. TURBINE 30,000 Kw.
Steam Pressure 1400 Lb. per sq. in. Temp. 925° F.
TURBINE 40,000 Kw. Topping
1937
1 BOILER 1,000,000 Lb. Steam per hr.
B.t.u. per Kw. Hr. 45,500 - Thermal Efficiency 7.5% Average 1917
" " " 19,700 - " " 17.35% " 1937

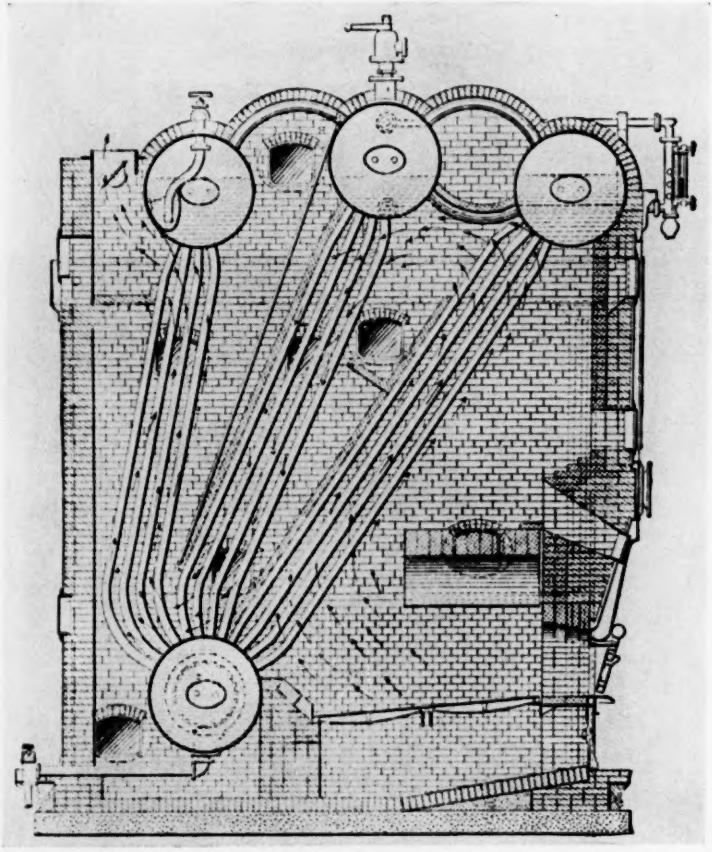
Comparison of steam generation in 1917 and 1937.

complished by increasing the steam pressures, final steam temperature, rates of heat transfer in the boiler, size of the boiler units and methods of burning the fuel.

In 1937 boilers were operating in the United States which had been designed to generate over one million pounds per boiler at 1400 lbs. per sq. in. steam pressure and 925° F. final temperature. The rates of heat transfer have increased. The walls of the modern boiler furnace are made up of closely

spaced tubes through which the boiler water circulates. A large percentage of the steam generated by the boiler is generated in this area by means of radiant heat at a very high rate of heat transfer.

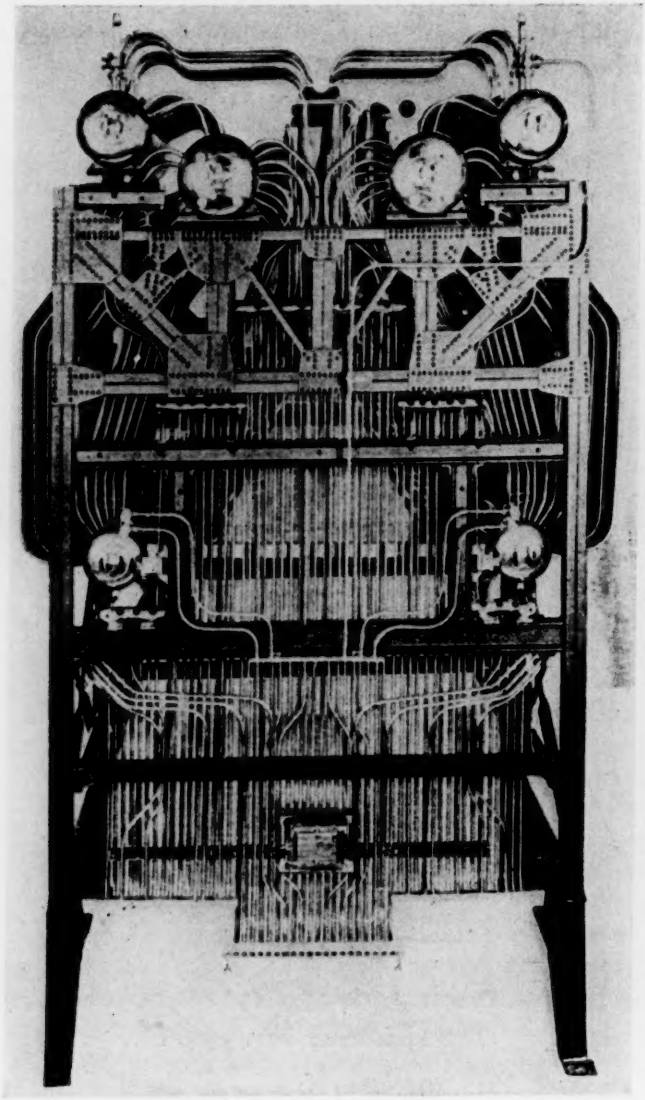
FIG. 2.



A type of boiler used in 1917.

Under these conditions of high rates of heat transfer and higher temperatures and pressures, the presence of scale—even of paper thickness—will result in tube loses with the resultant removal of the boiler from service. The modern

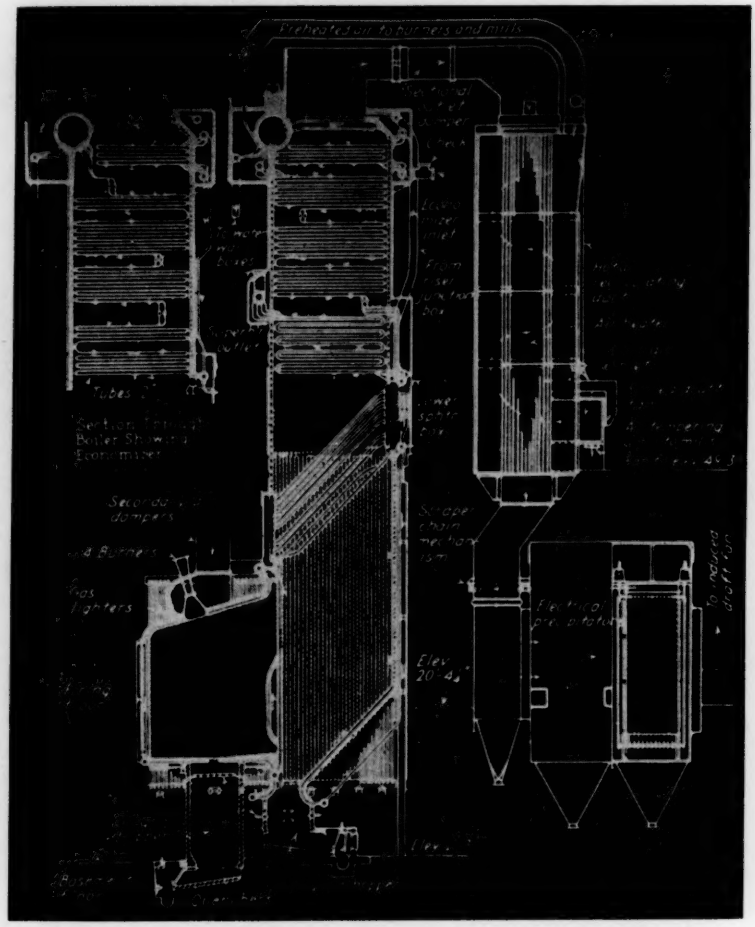
FIG. 3.



Arrangement of tubes and drums of the recently installed Conners Creek boiler is shown by this photograph of a model built by E. C. Keithley, Detroit representative of the Superheater Company, and presented as a gift to the Detroit Edison Company.

trend has been toward fewer and bigger boilers and consequently the unscheduled removal of one boiler from service results in a decrease of the station capacity.

FIG. 4.



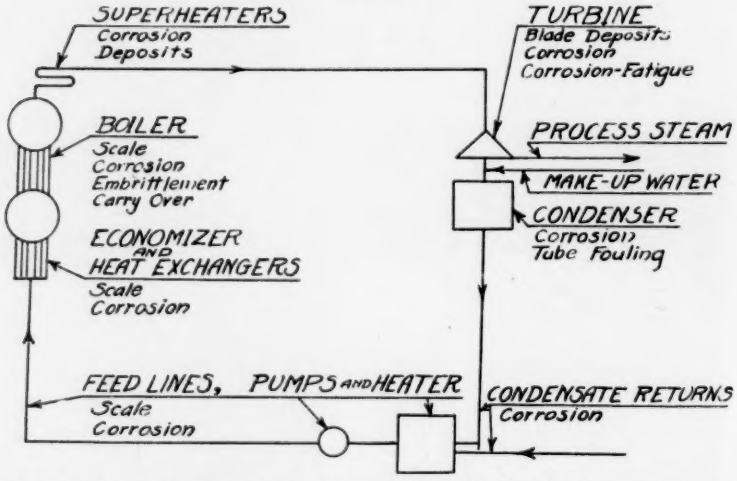
Steam generating unit: 412,500 lb. per hr. max. capacity at 1,200 lb., and 910° F.

With such emphasis being placed upon the availability of each boiler and its accessories attention has been focused on the purification or treatment of the boiler feed water so as

to prevent any difficulty which might arise in the water steam cycle of the steam power plant. Figures 1 to 4 illustrate the changes which have taken place in the steam power plants during the last twenty years.

Figure 5 shows a schematic diagram of the water-steam cycle of a steam power plant which will apply to central stations as well as industrial power plants. This diagram also enumerates the various types of difficulties commonly encountered in the water-steam cycle which must all be

FIG. 5.



Schematic diagram of powerplant showing types of difficulties which might be encountered.

considered in treatment of the feed water. Thus it might be a very simple problem to treat the water so as to prevent scale and corrosion in an economizer, but this treatment might produce a boiler water susceptable to carry over which in turn would cause deposits in the superheater or fouling of the turbine blade. In general the problem of treatment of the feed water in a steam power plant proves to be rather complex and a thorough study of all the conditions existing in the water steam cycle is essential before adopting any method of treatment. However, a few general principles covering the desired conditions have been evolved which serve as a guide in determining the type of treatment to be used. These may

be classified as follows:

1. Total Solids in the Boiler Water.
2. Steam Cleaning or Purification.
3. Gas Content of Feed Water or Steam.
4. pH Value throughout Cycle.
5. Removal of Scale or Sludge Forming Material Prior to the Boiler.
6. Internal Treatment for Prevention of Scale.
7. Prevention of Embrittlement.

1. TOTAL SOLIDS IN THE BOILER WATER.

It may be stated that a boiler water of low total solids will give a steam of high purity. Consequently a treatment which keeps the total solids low is desired in preference to one which allows the solids to increase to high values.

2. STEAM CLEANING OR PURIFICATION.

In many instances the demand for a high percentage makeup of treated water in steam boilers makes it necessary to have a boiler water containing relatively high total solids. In such instances it becomes necessary to clean the steam leaving the boiler. Since the steam leaving the boiler carries a small amount of entrained boiler water the total solids in the steam are proportional to the total solids in the boiler water and the percentage of the boiler water in the steam. By mechanical means it is possible to reduce the amount of boiler water carried into the steam to a low figure, however, if the total solids are high in the boiler water the resultant total solids in the steam becomes appreciable. Thus with a boiler water having 2,000 p.p.m. and 0.5 per cent. of this is in the steam as moisture there would be 10 p.p.m. in the steam. Contracts in many of the recent installations limit the total solids in the steam to less than 2 p.p.m. Thus it would be necessary to reduce the boiler water concentration to one-fifth of 2,000 or 400 p.p.m. to keep the total solids below 2 p.p.m. In order to reduce the total solids in the steam without reducing those in the boiler water methods of steam cleaning or purification have been developed. These methods in general make use of the principle of washing or scrubbing the steam with the water being fed to the boiler. If this

water enters the boilers at the temperature of the boiler water and is brought in intimate contact with the steam the boiler water in the steam will be replaced with feed water. Since the feed waters usually have a total solids content less than one-tenth of that of the boiler water the total solids in the resulting steam will be reduced to a low figure without changing those in the boiler water. In addition to reducing the total solids the composition of the solids is also changed. Thus a boiler water containing sodium hydroxide might give a steam having an appreciable sodium hydroxide content, but after being washed with a feed water having a low sodium hydroxide content the sodium hydroxide content of the steam might be reduced to nothing.

3. GAS CONTENT OF FEED WATER OR STEAM.

The chemicals dissolved or suspended in the boiler water are not the only impurities carried into the steam since gases present in the feed water or resulting from reactions taking place in the boiler water might be present. If oxygen is present in the feed water, it usually reacts with the steel in the economizers or boiler proper to produce corrosion, however, it frequently finds its way into the steam and causes corrosion in the superheaters and steam lines. Since oxygen reacts very rapidly with steel at boiler temperatures it is necessary to keep the oxygen content of the feed water as low as possible. This is accomplished by the use of deaerating equipment which boils the water in a properly vented steam atmosphere and thus reduces the oxygen content. However, it is often necessary to resort to the addition of chemical reducing agents which will react with the residual oxygen so as to remove it chemically from the cycle before it causes corrosion.

Other gases such as carbon dioxide, ammonia, hydrogen sulfide, sulfur dioxide, and volatile decomposition products of organic matter might also be found in the steam.

The carbon dioxide is usually present in the feed water combined with other chemicals such as sodium carbonate, calcium or magnesium bicarbonate or as free carbon dioxide. The higher temperature of the boiler water tends to decompose these chemicals with the subsequent liberation of the carbon dioxide. This gas tends to produce an acid steam

which often causes corrosion. Ammonia, however, tends to make the steam alkaline and is often considered desirable. However, at higher temperatures the ammonia might react with the various metals in the steam and water cycle to produce corrosion or weakening of the metals, consequently, its removal in the feed water is desirable. Hydrogen sulfide and organic sulfur compounds might cause corrosion as well as produce objectionable odors in the steam.

4. CONTROL pH VALUE IN THE CYCLE.

The term pH value is used in expressing the acidity or alkalinity of solutions. Thus a pH value of 7 indicates that the solution is neutral and a value greater than 7 indicates an alkaline solution while a value less than 7 predicts an acid solution. However, experience has shown that a neutral solution with a pH value of 7 will dissolve iron and produce corrosion. As the pH value of the solution increases above 7 the tendency to dissolve iron decreases. Since the reaction between the iron and solutions of various pH values are influenced by temperature and the salts in solution it is difficult to set a definite pH value for the entire water steam cycle. In some cycles it has been found that a pH value of 8 prevents corrosion (in the absence of O_2) in feed lines, heaters and economizers, whereas a pH value of 9.6 in the boiler proper is desired and a steam with a pH value of 6.5 has caused no trouble. In other cycles it is necessary to vary these values due to the presence of different chemicals in the cycle.

5. REMOVAL OF SCALE AND SLUDGE FORMING MATERIALS PRIOR TO THE BOILER.

The presence of calcium, magnesium, and silica in a feed water tends to produce scales or sludges of these materials in the feed lines, economizers or boilers. In many instances it is desirable to remove these impurities prior to admitting the water to the cycle. This is usually referred to as external water treatment. These impurities are reduced to low amounts depending on the methods of water softening used and this tends to reduce the tendency toward scale formation in the boiler.

6. MAINTENANCE TO PREVENT SCALE.

With the use of external softeners or evaporators there is a small residual amount of impurities left in the feed water and this is often augmented by contamination due to condenser leakage, etc. These impurities such as Ca, Mg, or SiO_2 concentrate in the boiler due to evaporation of the steam and will produce scale unless a chemical balance is maintained which causes their precipitation as a sludge. Thus the following illustrates one type of scale which might form under these conditions. Calcium sulfate in solution tends to form a scale on the heating surfaces of the boiler due to the fact that the calcium sulfate is appreciably soluble in the boiler water but decreases in solubility as the temperature increases. Thus this salt will decrease in solubility as the temperature is raised at the heating surface of the tube and form an adhering scale at this point. Calcium carbonate, however, is practically insoluble in the presence of small amounts of sodium hydroxide at boiler temperatures. Under these conditions the calcium carbonate would not tend to form scale at the heating surface of the tube. If the calcium could be kept as the carbonate in the boiler instead of the sulfate no calcium scale would form. The presence of a small amount of sodium carbonate tends to convert the calcium sulfate to the carbonate form as shown in the following reaction.

$$CaSO_4 + Na_2CO_3 \leftrightarrows CaCO_3 + Na_2SO_4.$$

This reaction may proceed in either direction depending on the relative concentration of the sodium carbonate and sodium sulfate. It has been shown that the presence of 30 p.p.m. of Na_2CO_3 will make the reaction proceed toward the formation of $CaCO_3$ and the prevention of sulfate scale. However, if the water contains less than this amount of Na_2CO_3 sulfate scale results.

Similar equilibria have been studied for the prevention of silica scale and it is possible to prevent silica scale by the precipitation of the silica as a non-scale forming calcium or magnesium silicate.

7. PREVENT EMBRITTLEMENT.

Embrittlement in steam boilers results in the failure of the steel in the riveted areas and the areas in the vicinity of

the rolled tube ends. This failure has been attributed to the action of the sodium hydroxide in the boiler water concentrating in the capillary spaces present in these areas and attacking the highly stressed boiler metal with the resultant cracking of the steel. Methods of chemical treatment have been worked out whereby the embrittling action of the water may be stopped. This has involved the maintenance of definite amounts of sulfates and chlorides in the lower pressure boilers and the reducing of the silica contents in the higher pressure boilers. Naturally the reducing of the sodium hydroxide content of the boiler water will tend to reduce the tendency toward embrittlement at all pressures. In the newer welded drums the riveted areas have been eliminated and the tube ends are the only remaining areas of potential danger. The methods of rolling the tube ends so as to keep the tube in contact with the drum or header surface on the inside tends to reduce the potential possibility of concentration of the boiler water in these areas.

CASE STUDIES OF FEED WATER PURIFICATION.

In order to obtain a clear picture of the subject of purification of feed water for the steam boiler it would be advisable to consider several cases which are representative of various types of boiler operation. Three different plants will be discussed. These will be as follows:

A. Central Station Electrical Generation.
B. Central Station Electrical Generation and Steam Heat.
C. Industrial Power Plant.

Case A. Central Station Electrical Generation.

The boilers in this station operate at 1250 lbs. per sq. in. steam pressure, and have economizers. The makeup is less than 3 per cent. of the steam generated or in other words 97 per cent. of the steam is condensed in surface condensers at a vacuum and returned to the boiler with the remaining 3 per cent. being water added to the cycle. The water available for use in the condensers and for make-up purposes is a surface water having a moderate bicarbonate hardness (140 p.p.m.) and a low sulfate hardness (30 p.p.m.).

In the central station evaporators may often be desirable since they use steam extracted from various stages of the turbine. This is also possible since the amount of makeup is very low. If the makeup water is evaporated and the distillate from the evaporator is properly vented it should be free from oxygen and very low in its carbon dioxide content which should give a water having a pH value of around 7.0. Since 97 per cent. of the steam is returned to the cycle, its only source of contamination would be through condenser leakage or contamination in surge or storage tanks. Since the condenser leakage would introduce the surface water in a quantity equal to less than 0.5 per cent. of the condensate passing through the condenser the total solids of the contaminated feed water would be 170 p.p.m. × 0.005 or 1.8 p.p.m. of hardness. This would not influence the pH value of the condensate, but it would introduce some oxygen. This might be largely removed by the vacuum in the condenser. However, in order to reduce the oxygen content to a minimum value a deaerating heater has been installed. In order to prevent corrosion from the residual oxygen the feed water is treated with small amounts of a reducing chemical such as ferrous hydroxide or finely divided iron. In order to prevent scale in the boiler a small amount of sodium phosphate is added to the boiler. The form of phosphate used (mono-di-tri-meta or pyro-) depends upon the alkalinity to be maintained in the boiler. Since the amount of scale forming material entering the system is small and the chemical treatment is very little the total solids are rather low. With a low total solids in the boiler water the blow down is very low. This also results in a low total

TABLE I.

Analyses of Raw, Evaporated, and Boiler Water from Case A—Central Station. Results in p.p.m.

	Hardness	pH	NaCl	SO_4 as Na_2SO_4	Total Dissolved Solids	SiO_2	Alkalinity as Na_2CO_3
Raw water	170	7.0	18	35	240	8	148
Evaporated	0	6.8	0	0	2	0	0
Boiler water	0	—	110	200	525	2	200

solids in the steam. However, due to the high pressure it is desirable to arrange to scrub or wash the steam with the feed water. Table I gives the analyses of the raw, evaporated, and boiler waters from this station.

Thus the treatment at this station is (1) evaporated makeup (3 per cent. maximum) with (2) small treatment of reducing chemicals for oxygen treatment, and (3) the use of a small amount of phosphate to prevent scale.

Case B. Central Station Electrical Generation and Steam Heat.

In steam heating stations the makeup is often very high due to the fact that the condensed steam is not returned to the station. The water treatment in use at the Rochester Gas and Electric Corporation's new addition to Station No. 3 at Rochester, New York, will serve to illustrate the chemical treatment in use for feed water to a 700 lbs. per sq. in. steam boiler using 95 per cent. makeup.[2] The water comes from the Genesee River and is treated in a cold process softener with lime soda ash and a coagulant to clear the water and soften it to a residual hardness of less than 20 p.p.m. The clear but not filtered effluent from this softener is pumped to a Cochrane hot-process softener. The softener also serves as a deaerator. Disodiumphosphate is added to the softener to reduce the hardness to zero. The water is filtered through anthracite coal (anthrafilt) and pumped to the boilers. In order to prevent precipitation of calcium phosphate in the feed lines sulfuric acid is added after the filter in order to decrease the pH value to 8.8. At this pH value the calcium phosphate will not form precipitates in the feed lines or pumps. The feed water is also treated with sodium sulfite so as to remove the last trace of oxygen and prevent corrosion. The concentration in the boiler water is necessarily maintained high so as to reduce the boiler blow down to a low value. Table II gives typical analyses of raw, treated, and boiler water from this station.

In order to be certain that the boilers would deliver clean steam with such high concentrations of soluble solids in the boiler water a steam washer of the bubble type was installed.

[2] "Boiler Feed Water Treatment at a High-Pressure Station." S. T. Powell, I. G. McChesney and F. Henry. *Ind. and Eng. Chem.*, **30**, 400–406 (1938).

TABLE II.

Typical Analyses of Raw and Treated Waters at Station 3 (in Parts per Million).

Sampling Point	Alkalinity		pH	NaCl	Total Sulfate as Na_2SO_4	PO_4	Hardness	Total Dissolved Solids	Turbidity
	Phenolphthalein	Methyl orange							
Raw water, Genesee River	0	116	7.4	94	89	—	160	344	55
Scaife plant effluent (clear well)	42	78	9.7	110	106	—	15	309	5
Cochrane softener effluent	42	76	10.0	105	100	7.4	4	252	25
Pressure filter effluent	39	72	9.9	103	95	3.9	0	286	0
Boiler feed water after acid treatment and addition of condensate	7	34	8.9	86	110	3.0	0	230	0
Boiler blowdown	272	352	11.6	900	2400	62	0	3711	0

The station is provided with a modern and complete chemical laboratory which is operated 24 hours a day on three shifts. Sampling coils for boiler water and boiler feed water are installed in the laboratory and samples are drawn every 4 hours. Determinations are made for alkalinity, hardness, phosphate, sulfate, chloride and pH.

A continuous recording pH indicator records the pH value of the acid treated feed water and other recording instruments make continuous records of the conductivity of the steam as well as of the boiler water.

A summary of the chemical treatment at this station is as follows:

1. Preliminary continuous cold process lime and soda ash softening.
2. Secondary hot process phosphate softening.
3. Acid treatment to reduce pH value and prevent phosphate fed line scale.
4. Sodium sulfite treatment to remove last trace of oxygen.

Case C. Industrial Power Plant.

An industrial power plant recently installed in a paper mill will serve to illustrate this type of power plant. The mill had been operating with a steam pressure of 160 lbs. per sq. in. with a low pressure turbine and process steam being used in the mill. The condensate returns were around 40 per cent. with river water makeup for the boilers. New

boilers with a steam pressure of 700 lbs. were superimposed on the low pressure plant. This meant a 60 per cent. makeup to a boiler operating at 700 lbs. pressure. The water available is clear filtered river water having the total hardness equal to 175 p.p.m.

In order to remove the hardness and keep the alkalinity low in this boiler water the combination hydrogen and sodium zeo-karb method of treatment was used. The zeo-karb or carbonaceous zeolites are capable of being regenerated with either sodium chloride or sulfuric acid. If this type of zeolite is regenerated with sodium chloride the resulting softening cycle furnishes the sodium salts of the calcium or magnesium salts present in the water. Thus the calcium and magnesium bicarbonates are converted to the sodium bicarbonates and the calcium and magnesium sulfates are changed to sodium sulfate. The sodium sulfates and chlorides are unchanged. When the zeo-karb is regenerated with sulfuric acid it is referred to a H_2Z to distinguish it from the one regenerated with sodium chloride which is called NaZ. The H_2Z will react with the calcium and magnesium bicarbonates to form carbonic acid (H_2CO_2) and with the chlorides and sulfates to form hydrochloric or sulfuric acid. The effluent from this type of softener is distinctly acid and is not suitable for boiler feed purposes. However, if part of the raw water is passed through NaZ, the rest through H_2Z, and the two mixed in proper proportions the alkalinity of the NaZ effluent neutralizes the acidity of the H_2Z effluent. The carbonic acid may be readily decomposed to water and carbon dioxide by passing air through the mixed solutions (degasified). Any desired residual alkalinity may be obtained as a result of this treatment and there will be a marked reduction in total solids.

Thus Table III illustrates how the total solids has been reduced from 210 in the raw water to 96 in the mixed degasified effluent from the NaZ and H_2Z softeners. The hardness has been reduced to a maximum value of 2 p.p.m. After the mixed degasified waters pass through the deaerating heater where the oxygen is removed and the residual sodium bicarbonate breaks down to form sodium carbonate the pH value of the feed water is 9.0.

TABLE III.

Analyses of Raw, Treated, and Boiler Water from Case C—Industrial Power Plant. Results in p.p.m.

	Hardness	pH	NaCl	SO_4 as Na_2SO_4	Total Dissolved Solids	Na Alkalinity as $CaCO_3$	SiO_2	Free Mineral Acidity
Raw water	175	6.9	20	45	210	0	9	—
H_2 zeo-karb effluent	2	—	—	—	56	0	9	45
Na zeo-karb effluent	2	8.3	20	45	212	135	9	—
Mixed zeo-karb effluent after degasifier	2	6.8	20	45	96	20	9	—
Boiler water	0	11.5	300	675	1275	300	2	—

In order to protect the feed lines from corrosion and to aid in removing the silica from solution in the boiler finely divided iron is added to the feed water. The iron reacts with the residual oxygen to remove it from the water and aids in preventing silica scale in the boiler.

These boilers also have steam scrubbers installed so as to aid in purifying the steam.

A summary of the chemical treatment at this industrial plant is as follows:

1. Combination NaZ and H_2Z treatment.
2. Iron treatment for oxygen and silica removal.

There are many other methods of water treatment which may be used in treating boiler feed waters, however, when considering the use of any method of water treatment the possible effect of the treatment on all portions of the water steam cycle of the power plant must receive serious consideration.

TIME VARIATIONS OF COSMIC RAYS.*

BY

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One of the characteristic features of cosmic rays is their approximate constancy. Day and night, year after year, the rays come flowing into the earth with almost uniform strength. Similarly we find that the cosmic rays are distributed uniformly over the earth, except as their intensity is affected by the earth's magnetic field. Because of this uniformity, the hypothesis has arisen that the rays come equally from all parts of space. Alternative hypotheses, however, would place the origin of these rays in certain definite locations, such as novæ, or in the galactic plane. One of the major reasons for studying the time variations of cosmic rays is thus to distinguish between such theories regarding their place of origin.

Refined observations show various types of minor time variations of cosmic rays. With the gradual development of instrumental technique, we may suppose that of the recent measurements few are appreciably affected by instrumental error. Much more disturbing in modern work are the statistical fluctuations of the cosmic rays themselves. These arise from the fact that the number of the cosmic rays observable in a short interval is limited, so that in a given time interval more or less cosmic rays than normal may enter the measuring instrument. There are also such phenomena as showers and bursts of rays giving great effects over short intervals of time, and producing changes of a surprisingly large order of magnitude. By continuing the readings, however, over a sufficiently long period, deviations of this type may be averaged over to such an extent that they do not seriously affect the data.

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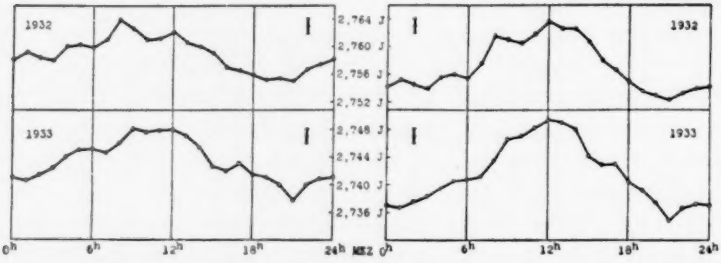
* Based upon a paper read at the Chicago Cosmic Ray Symposium, June, 1938.

Bothersome also is the effect due to changes in the barometric pressure, that is, changes in thickness of the atmospheric layer through which the rays must come before they reach the earth. When measured at sea-level there occurs a decrease of about 2 per cent. for each centimeter rise of barometric pressure. When we recall that the pressure at a given station may vary by two or three cm., this means a considerable change in the strength of the cosmic rays. Though the origin of this effect is well known, it is not a simple matter to apply the necessary corrections in a satisfactory manner.

PERIODIC EFFECTS.

After instrumental, statistical and barometric fluctuations have been taken into account, there remain other measurable changes in the cosmic rays. Some are of a periodic, others of a non-periodic type. The recognized or suspected periodic variations are those which follow the solar day, the sidereal day, the annual period, that is, seasonal fluctuations, and the period of the sun's rotation.

FIG. 1.

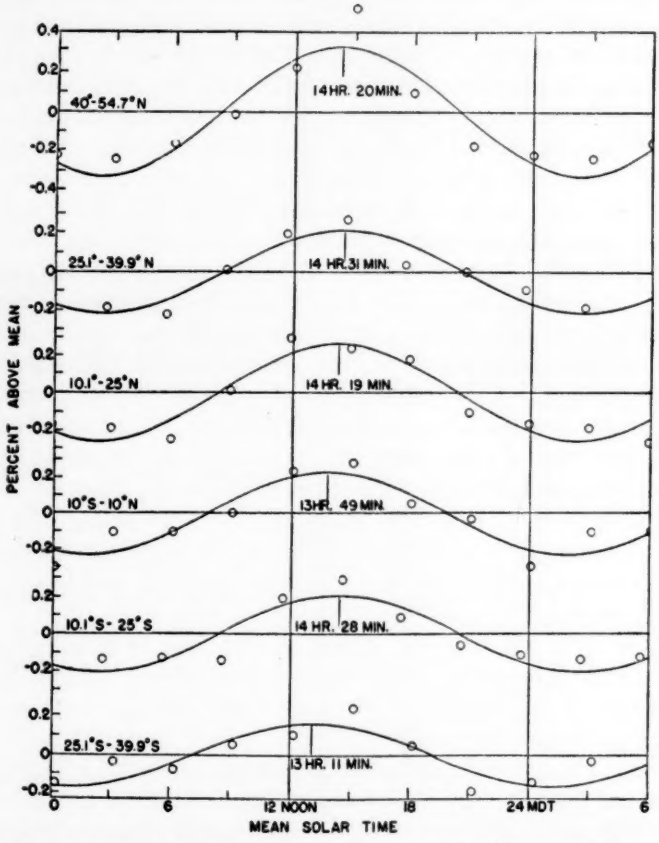


Solar time variation of cosmic rays, showing maximum near noon. Left, corrected for barometer effect. Right, corrected for changes of both barometer and atmospheric temperature (Hess, Graziadei, Steinmaurer).

Of these, the variations following the solar day are the best established. Thus Fig. 1 shows the data obtained by Hess and his collaborators [1] during the years 1932 and 1933, on the Hafelekar, in Austria. The measurements were made with a Steinke cosmic ray meter, shielded with 10 cm. of lead. Two sets of curves are given, one of which shows only the data corrected for the barometer effect, the other using data to which have been applied also a correction for the

variation of the temperature of the outer air. Whichever is considered the more reliable, there is evidently a maximum of intensity near noon, and a minimum not far from midnight. Similarly definite results have been published by Forbush [2] and others.[3]

FIG. 2.



Solar time variations of cosmic rays at different latitudes on the Pacific Ocean (Thompson).

Julian Thomson [4] has recently analyzed the data which Turner and I have collected [5] over a period of about a year on the Pacific Ocean en route between Vancouver and Sydney. These data have enabled him to determine the diurnal variation at different latitudes. The results of his analysis are shown in Fig. 2. The data over different latitude ranges

have been so interpreted as to give the first harmonic of the 24 hour curve. In the figure the resulting curve as calculated for each latitude range is compared with the data expressed as tri-hourly means. Perhaps the most noteworthy feature is that the magnitude of the daily variation here shown, when expressed in per cent. of the total intensity, is almost independent of the latitude. The amplitude of this 24 hour period is about the same as that indicated by the data of Hess.

Do these daily variations of the cosmic rays mean that part of them come directly from the sun, or is this some secondary effect of the sun's radiation striking the earth? Hess [6] has defended the former view, pointing out that the diurnal effect can be accounted for if as much as 0.5 per cent. of the total cosmic ray beam at an altitude of 2500 meters comes directly from the sun. By comparing the magnitude of the effect with and without the 10 cm. lead shields, he calculated that this solar component should be of at least as great penetrating power as the main part of the radiation. Most students of the phenomenon now favor, however, the view that the effect is a secondary one. The surprisingly large effect on cosmic rays due to magnetic storms supports Gunn's suggestion [7] that this diurnal variation may be caused by changes in the earth's magnetic field following the solar time. This view is strengthened by the fact that the cosmic ray changes following the earth's magnetic field are also nearly independent of latitude (Fig. 3, below).

The type of periodic effect which has been of greatest theoretical interest is the one which follows sidereal time. Suggestions by Nernst that cosmic rays come from certain types of stars led Kolhörster and others to investigate this effect. The early indications that cosmic rays are a maximum when the Milky Way or certain of the fixed stars are overhead have not been confirmed by more recent observations.[8] A few years ago, however, Dr. Getting and I called attention [9] to the fact that, due to the earth's rapid motion with the sun following the rotation of the Milky Way, there should be a greater intensity of the cosmic rays on the front of the earth than on the back. This is toward the constellation Cygnus, about 47 degrees north.

If we were to neglect the effect of the earth's magnetic

field, because of this motion the rays should be of about 0.8 per cent. greater intensity on one side of the earth than on the other. When we take into account the fact that the rays are not coming perpendicular to the equator, and that most of the measurements are made in temperate zones, purely vector considerations reduce this 0.8 to an anticipated 0.4 per cent., or an amplitude of the first harmonic of 0.2 per cent. A very recent calculation by Vallarta, Graef and Kusaka [10] shows that for rays striking the earth vertically at the equator, instead of an amplitude of 0.3 per cent. to be expected without a magnetic field, with the magnetic field an amplitude of from 0.2 to − 0.06 may be calculated, according to the relative number of positive and negative particles in the primary cosmic rays. The time of maximum should lie between 8 hr. 40 min. and 13 hr. 20 min. unless very heavy shielding against the lower energy rays is used. At higher latitudes it is probable that the maximum should occur closer to the time 20 hr. 40 min. when the meridian is in the direction of the sun's motion. The exact calculation for the higher latitudes has, however, not been carried through.

The following table gives a summary of the available data which are sufficiently precise to be significant. Assigning

TABLE I.

Sidereal Time Variations of Cosmic Rays.

Observer.	Latitude.	Geomag. Latitude.	Barometer.	Date.	Amplitude Per Cent.	Time of Maximum.
1. Illing [11] (Hess). .	47° N.	49° N.	580 mm.	1932	.015	2 hr.
				1933	.006	13
				1934	.045	20
2. Schonland, Delatizky, Gashell [12]. . . .	34° S.	32° S.	760	1933	.05	23
				1934	0	—
				1935	.06	0.5
3. Forbush [13].	39° N.	47° N.	760	1935–37	.03	22
4. Forbush [13].	12° S.	1° S.	515	1937	.06	5
5. Thompson [14]. . . .	49° N. to 34° S.	55° N. to 43° S.	760	1936	0	—
6. Gill [15].	19° N.	29° N.	585	1937–38	.06	18

equal weight to each year's data at every station, we obtain an average of 0.018 ± 0.005 per cent., with a maximum at 23 hr. This is considerably smaller than might be anticipated from the theoretical calculations, and is not in good agreement as to phase. If, however, we consider only the data from the higher latitudes, excluding those from stations 4 and 5, the average rises to 0.033 ± 0.006 per cent., with the maximum at 21.5 hr. Such a selection is suggested by the large calculated effect of the earth's magnetic field at the equator, indicating that the amplitudes and phases there observed may not be comparable with those at other latitudes. The departures in phase are, however, in the direction to be expected if the incoming cosmic ray particles are predominantly negative rather than positive, as is indicated by the directional experiments. Though the internal consistency of the data from each station is hardly great enough to show the reality of the sidereal variation, there would thus appear to be accumulating enough evidence from all sources to indicate rather clearly the presence of a small diurnal variation. Whether this can be identified with the effect predicted as due to rotation of the galaxy cannot be answered definitely until the detailed calculations of the effect of the earth's magnetic field at the higher latitudes have been completed.

Counter observations by Barnothy and Forro,[16] and more recently by Getting,[17] have indicated a sidereal time effect much closer to the predicted value. The phase of the effect observed by Barnothy and Forro varied so rapidly with shielding, however, that the phenomenon which they were observing could hardly be identified with the galactic rotation effect now under discussion. Getting's experiments extended over so short a part of the year that it is not yet possible to say whether his observations represented a sidereal or a solar time variation.

Even though the cosmic rays should form a part of our galactic system, sharing the rotation about the nucleus of the Milky Way, there should yet remain an effect caused by the motion of the sun relative to the stars in our neighborhood. This motion is only about 7 per cent. as fast as that of the rotational motion which the sun shares with the other stars. The effect on the cosmic rays should be correspondingly

smaller, certainly not greater than 0.015 per cent. Its maximum should occur about 2 hours earlier than that due to the galactic rotation. If the incoming cosmic rays were predominantly negative, this prediction would be in reasonable accord with the average of all the experimental data.

At the present stage of the study, therefore, it is not possible to decide whether the very small but apparently real sidereal time variation is or is not ascribable to motion of the earth with the rotation of the galaxy.[18] Both the experimental and the theoretical aspects of the question need to be investigated in greater detail. It would appear, however, that a continuation of the studies now in hand should enable us to reach a definite answer.

Mention should be made of certain other periodic effects. There is good evidence for the existence of seasonal variations. Indications of a maximum of intensity during the cold months have been presented by Hess [19] and by Compton and Turner.[5] This conclusion is confirmed by a new analysis made [20] by Mr. Gill and Mrs. Ardis Monk of data received from various stations during the past two years.

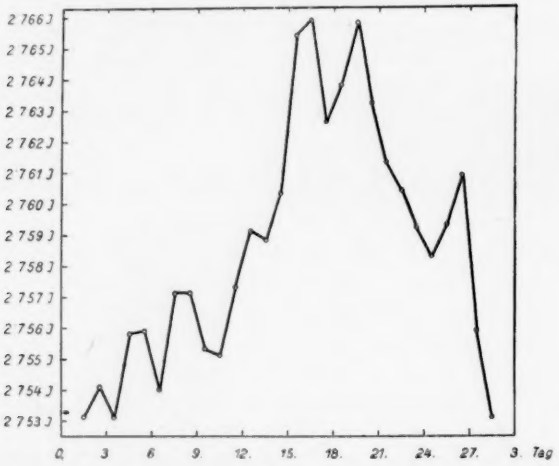
It is possible to interpret this seasonal variation as a temperature coefficient of the cosmic ray intensity, since it is found [5] that in certain cases at least a close correlation occurs between the cosmic ray intensity and the daily mean atmospheric temperature. At different locations the "temperature coefficients" thus calculated from the seasonal variations are from -0.05 to -0.3 per cent. per degree C.

Blackett [21] has recently suggested an explanation of this temperature coefficient, based upon the theory that the mesotrons which form the penetrating component measured by the cosmic ray meters are secondary particles which have a natural life comparable with the time required to traverse the atmosphere. Thus expansion of the atmosphere with increasing temperature will increase the time of passage of the mesotrons, and decrease the probability of their reaching sea level. On this basis, Blackett calculates a temperature coefficient of about 0.2 per cent., in agreement with observation. Proof of the secondary origin of the mesotrons [22] has given support to this theory, as has also evidence brought forward by Gill [20] supporting Blackett's prediction that near the

equator the temperature coefficient should be less than in temperate zones.

A notable conclusion from the study of the seasonal effects is that there is no general increase over the earth when the earth is at perihelion on January 1. From the inverse square law, an annual change of 6 per cent. would be anticipated for any solar component. The evidence does not rule out the suggestion of Hess of a very small component of 0.5 per cent. of the total to account for the solar diurnal variation; but it does show that rays from the sun do not play a major role in cosmic ray phenomena.

FIG. 3.



Mean intensity of cosmic rays throughout a cycle of 27.2 days (solar rotation), average of 45 cycles (Graziadei).

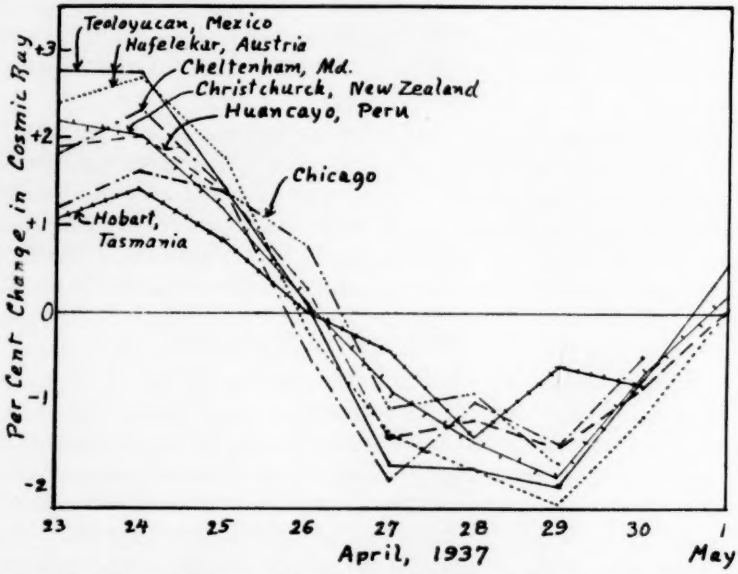
Graziadei has also shown [23] that the cosmic ray intensity has a periodicity of about 27 days. Figure 3 shows the variation over a complete cycle of 27.2 days, taken as the average of 45 successive periods. His result has been confirmed by independent calculations of Gill.[20] The amplitude of this monthly oscillation is about 0.2 per cent. Graziadei relates this effect to the solar rotation, since a corresponding effect is recognized in terrestrial magnetism. It should be noted, however, that the period is likewise within experimental error equal to that of the revolution of the moon about the earth.

If the effect is indeed caused by the sun, its presence may be taken as a support of Vallarta's hypothesis of the effect of the sun's magnetic field.

NON-PERIODIC EFFECTS.

The correlation which is found at many stations between cosmic ray ionization and the temperature of the outside air has been mentioned above. This connection was first emphasized by Messerschmidt.[24] Observers in different locations

FIG. 4.



Changes in cosmic ray intensity at various stations associated with the magnetic storm of April, 1937, showing the world-wide character of the effect.

and at different times find widely differing correlation factors. The temperature coefficients are usually found to be positive for daily temperature changes, and negative for seasonal changes. Here is a paradox which awaits solution.

That a correlation exists between cosmic ray intensity and the changes of the earth's magnetic field associated with magnetic storms was observed by Corlin in 1931, and later discussed by him in some detail.[25] With the striking evidence of the world wide character of these changes as presented last

year by Forbush,[26] their importance has become more evident. I shall content myself with showing a single figure which demonstrates this world wide character. It will be noted that while from geomagnetic latitude 47° N. (Cheltenham) to 43° S. (Christchurch) the percentage variation is nearly unchanged, at Chicago (52° N.) and Hobart (52° S.) the change is distinctly less, but is yet very measurable. Corlin records changes of the cosmic rays as far north as Abisko (66° N.) which show the same general features. The presence of this effect must mean that even at latitudes of 66° some rays capable of penetrating the atmosphere are prevented by the earth's magnetic field from reaching the earth. Theoretically one would anticipate no change at latitudes above the knee of the latitude-effect curve.

There remain well established non-periodic variations of unknown origin, which are described as "of the second kind." Some of these are doubtless associated with changes in the earth's magnetic field. It is not clear, however, that they are all explicable in this manner. At present these fluctuations merely mask the other changes that we desire to study.

LOCATING THE SOURCE OF COSMIC RAYS.

The outstanding result of these observations is the high degree of uniformity of the radiation from all directions. It was this fact which led Hess in 1913 and Millikan in 1925 to stress their cosmic origin. For where, outside of our own atmosphere, can an isotropic source be found unless we look beyond our galaxy? Thus the idea of an extra-galactic origin of the rays arose. Yet, as we have seen, the most recent measurements indicate a spherical symmetry of the radiation that is so exact as to make even this explanation difficult. We should have expected the rotation of the Milky Way to have produced an asymmetry greater than the observations show. Unless further refinements of the theory lead to a prediction of a smaller effect than would now appear probable, it would appear that we must consider the cosmic rays as relatively local. Just how local depends upon the magnitude of the effect revealed by further observations which may be ascribed to the motion of the sun among the neighboring stars.

In the paper in which he first announced the existence of

rays from outside the atmosphere,[27] Hess noted that, "if one is thinking of a direct gamma radiation propagating itself in straight lines," their spherical symmetry must imply a cosmic rather than a localized origin. Now that the evidence is clear that the rays are not of the gamma ray type, we must re-examine the possibility that the cosmic rays even in remote space do not proceed in straight lines. Last year Dr. Chau and I published [28] a letter in which we called attention to the fact that if stars or galaxies have magnetic fields sufficiently powerful to deflect the rays, certain serious difficulties connected with their large energy may be avoided. At the Notre Dame symposium last month I outlined all too briefly Alfven's yet unpublished theory [29] which shows that such strong stellar and galactic magnetic fields are very probable. We must accordingly consider with care the suggestion that the isotropic character of the rays is the result of many random deflections within a distance from the earth that is small compared with the diameter of the galaxy.

First, let us repeat in slightly modified form Alfven's calculation of the magnetic field of a rotating galaxy. He starts with the provisional assumptions: 1, that all of the cosmic ray particles are positively charged, and 2, that the cosmic ray particles do not share the galactic rotation. He then notes that in order to avoid an impossibly large space charge, an approximately equal number of negative ions must be present in interstellar space, drawn perhaps from the stellar atmospheres. Such slow negative ions, because of collision with interstellar matter and with the magnetic fields of the stars, will, however, share the galactic rotation. The positive cosmic ray particles will thus move in a magnetic field produced by the rotation of the negative ions with the galaxy. His calculation, which I have checked by another method, gives as the strength of this field about 10^{-1} gauss. For cosmic ray particles of 10^{10} ev. energy, this would mean a radius of curvature of the order of the radius of the earth. Obviously, the provisional assumption of positive particles uninfluenced by the galactic rotation is untenable.

The assumption of exact equality of positive and negative cosmic ray particles is made difficult by the observation that the cosmic rays near the equator come mostly from the west.

Escape from this difficulty might be possible if the absorption of the positive cosmic ray particles in the atmosphere were less than that of the negative particles. In this case the westerly predominance would not necessarily indicate an excess of incoming positives, but merely a more rapid absorption of the incoming negatives. There is a suggestion in some of the cloud chamber experiments of Anderson and Auger that such a difference may exist, but the results are tentative. If such a difference in absorption is assumed, it will imply some, even though small, difference in the number of positive and negative cosmic ray primaries after they have been coursing through space for a long time. Thus the assumption of an exact equality of positive and negative particles would not seem to be permissible.

If we suppose that the magnetic field of the sun at its surface is of the order of 30 gauss, it can be shown that the ordinary thermo- and photo-electrons within a radius of about 5 times that of the solar system will be controlled by its magnetic field. Because of their greater mass the positive ions of equal energy will not be held by the magnetic field of the sun except within a much smaller volume. If the excess of negative electrons following the sun within this region is no greater than the space density of the cosmic rays, the magnetic field resulting from their motion with the sun will be a determining factor on 10^{10} ev. particles that come within a distance of from 10^{-1} to 10^{-2} light year. This may be taken as a rough estimate of the collision cross sectional radius of a star for an approaching cosmic ray.

These conditions suggest that cosmic rays may move some hundreds of light years between collisions with stellar magnetic fields, but that the magnetic field of the galaxy itself may present an almost impassable barrier against their escape. This affords just the situation needed to supply electrical particles without associated photons. For whatever the origin of the rays, the associated photons would promptly escape from the galaxy just as starlight does, leaving the electrical particles behind.

If we suppose with Lemaitre that the cosmic rays originated with the primeval explosion of the universe, the electrical particles initially in each galaxy may thus have remained

there almost undiminished in numbers. Also, since no red shift would be anticipated for rays kept within the galaxy, we are relieved from the necessity of supposing that early in the world's history their individual energies greatly exceeded the stupendous values that we now observe. In fact, the occasional collisions of the cosmic ray particles with the magnetic fields of moving stars would gradually increase their energy.

Most significant, however, is the fact that if the cosmic rays are a part of our galaxy the power required to produce them is not prohibitively great. The calculations of Millikan and Regener have indicated that the energy received by the earth as cosmic rays is about the same as that received as starlight. If this were a fair sample of the cosmic rays in all space, since starlight is chiefly confined to the galaxies, this would mean that cosmic ray energy is being produced many times more abundantly than is light energy. No adequate source for such a colossal flow of power has appeared. If, on the other hand, the cosmic rays are confined to our galaxy, whereas starlight escapes from it freely, the cosmic radiation may be the accumulation over an immensely great period of such high speed particles as have been gradually produced. Thus the source of cosmic ray power would be vastly feebler than the source of starlight.

We should on this view continue to think of the cosmic rays as very old, perhaps dating from the origin of the galaxy itself, perhaps the accumulation of millions of years of high energy particles gradually produced by some generating process within the galaxy. But we should not think of them coming from space which on an astronomical scale is very remote.

It would thus appear that the theoretical difficulties in accounting for the origin of the cosmic rays are less severe if they are supposed to be a part of our own galaxy than if we suppose them to come from outside. The observations of the very small sidereal time variations seem also to support this view, though further work is required before any definite conclusion to this effect is justified.

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DISCUSSION ON: "TIME VARIATIONS OF COSMIC RAYS" AND SUMMARY OF DAY'S DISCUSSIONS, BY A. H. COMPTON.

Dr. W. F. G. Swann commented as follows:

Of course, I think we must regard the elements underlying the assumptions of a rotation of the space charge with the galaxy as rather dubious. Professor Compton has emphasized the evidence which exists for supposing that the assumption of a magnetic field of the order of one tenth of a gauss in the galaxy and such as has been calculated on the assumption of complete rotation of the space charge is inconsistent with the observations of cosmic-ray phenomena. I should like to call attention to one or two other remarkable consequences of the existence of fields comparable with one-tenth gauss throughout the galaxy. Such a galaxy would constitute a magnetic shell and would attract another one. I find that if our galaxy had a field of one-tenth gauss at its center, it would have a moment of $10^{-2}R^3$, where R is its radius. Two such galaxies separated by 200,000 light years would, if properly oriented, attract one another. By taking our galaxy as weighing 10^{42} grams, I find that under such an influence it could acquire one-tenth of the velocity of light in respect to the other galaxy in 2×10^5 years. Such considerations, even with much smaller magnetic fields, have interesting consequences concerned with the relative motions of the galaxies.

Another matter concerns the electric moment of such a galaxy. In relation to a point, a moving magnetic galaxy would have an electric moment of the order v/c times its magnetic moment; and if v/c were comparable with 0.1, it is easy to show that, observed from a frame of reference in which the galaxy was moving with the aforesaid velocity, it would produce, at points which were at distances from its center comparable with radius, a potential of the order 10^{22} volts. Of course, the forces arising from this electric field are in part another aspect of the forces produced on moving charges by the magnetic field in the frame of reference of one who moves with the galaxy. However, to one in whose frame the galaxy is moving with the velocity v, the forces concerned would really be responsible for the acquirement of energy by charged particles in terms of measurements made in this fixed frame of reference.

Coscos Tatti—The Last Word in Indian Air Conditioning.—(*Heating and Ventilating*, Vol. 35, No. 11.) Coscos Tatti may not be protected under any American patents, but they have been the means of bringing to the sultry gentry of India for ages past a form of air conditioning that is effectual, although primitive, according to J. A. DOOLEY, manager of the refrigeration department of the International General Electric Co. (India), Ltd., Bombay. The instruments of comfort bearing this strange name are window screens of woven grasses over which water is sloshed at regular intervals. They provide cooling and an additional refinement which is lacking in modern air conditioning—they add a delicate fragrance to the air. As the water evaporates from the screens it cools the air and releases the scent of the grasses, which, says Mr. Dooley, is like the sweet smell of the earth after an early summer rain. Prevailing winds and breezes take the place of fans in providing ventilation and sufficient air for evaporation, and consequently only rooms on the windward side of the house can be cooled. In older times slaves were accustomed to pour buckets of water on the screens, but this has given way to an automatic system, consisting of a trough at the top of the screen. Water runs slowly into the trough, and when it is filled to a given point, it becomes overbalanced and dumps its load over the screen. Counterweights bring the trough back to its original position ready to be filled.

R. H. O.

ON THE FUNDAMENTAL PRINCIPLES UNDERLYING THE THEORY OF COSMIC RADIATION.[1]

BY

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The recent extensive development of the theory of the charged component of cosmic radiation on the part of Lemaître and Vallarta and their collaborators, together with the experimental confirmation of certain aspects of the theory by a number of investigators, have led to a widespread interest among physicists for a clear understanding of the fundamental principles underlying the Lemaître-Vallarta theory. The present paper contains a brief exposition of such fundamental principles including, for the sake of completeness, the derivation of the equations of motion of a charged particle in the field of a magnetic dipole; a brief discussion of the elegant application which Lemaître and Vallarta made of Liouville's theorem; and finally, a detailed account of Störmer's fundamental contributions dealing with the normalization of the equations of motion and the conformal transformation of the meridian plane.

The theory of the charged component of cosmic radiation, which in the last six years has been the subject of extensive investigations on the part of Lemaître and Vallarta and their collaborators, is based essentially on the fundamental principles governing the motion of charged particles in the magnetic field of the earth and on the simple hypothesis of the existence, at large distances from the earth, of a homogeneous and isotropic distribution of charged particles.

[1] Contains essentially the material of the Appendix in the author's Ph. D. dissertation, "Calculation of a Family of Asymptotic Orbits in the Field of a Magnetic Dipole," submitted to the Massachusetts Institute of Technology in partial fulfilment of the requirements for the degree (1938). This material in turn represents a translation of an article by the writer, "Las ecuaciones del movimiento en la teoría de la radiación cósmica," published by the National University of Mexico, in *Ingenieria*, Vol. XI, No. 4 (1937).

Lamaître and Vallarta [2] have shown that, by virtue of the earth's magnetic field, cosmic rays of a given energy do not reach all points of the earth. In general, at a given magnetic latitude, charged particles of a given kinetic energy are capable of reaching the point in question only along certain allowed directions, the integrated totality of which fills a cone of more or less complex form. Furthermore, as a corollary to Liouville's theorem, it is concluded that the intensity of cosmic radiation per unit solid angle within the cone of allowed directions, for a given energy, is the same as the intensity at large distances from the earth.

In the theory of the charged component of cosmic radiation, therefore, the fundamental problem consists in the determination of the boundary of the cone of allowed directions whose generators are the trajectories separating orbits coming from infinity from orbits coming from another point of the earth. In this manner Lemaître and Vallarta have developed the theory which carries their names and which fully accounts for such experimental results as: (1) the variation of total intensity as function of magnetic latitude, originally discovered by Clay [3] and verified independently by Compton and his collaborators; [4] (2) the azimuthal asymmetries of cosmic radiation, particularly in the meridian plane (North–South) and in the East–West vertical plane, observed experimentally by Compton [5] and independently by Johnson; [6] (3) the so-called longitude effect, observed experimentally by Clay and his assistants [7] and independently by Millikan and Neher,[8] and which Vallarta [9] has shown arises from the fact that the equivalent magnetic dipole, which represents the earth's magnetic field to a very close first approximation, does not coincide with the geographic center of the earth.

The theory of cosmic radiation, as developed by Lemaître

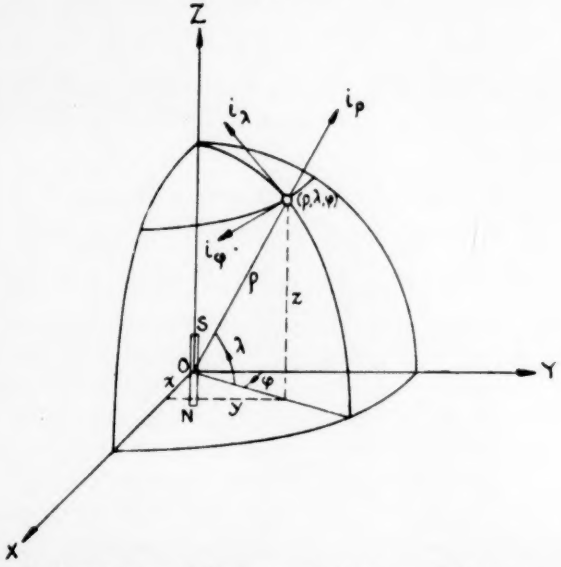
[2] *Phys. Rev.*, **43**, 87–91 (1933).
[3] J. Clay and H. P. Berlage, *Naturwiss.*, **20**, 687–8 (1932).
[4] A. H. Compton, *Phys. Rev.*, **41**, 111–3 (L), (1932).
[5] A. H. Compton, *Phys. Rev.*, **43**, 397–403 (1933).
[6] T. H. Johnson, *Phys. Rev.*, **43**, 835–6 (L), (1933).
[7] J. Clay et al, *Physica*, **1**, 829–838 (1934).
[8] R. A. Millikan and H. V. Neher, *Phys. Rev.*, **47**, 205–8 (1935).
[9] M. S. Vallarta, *Phys. Rev.*, **47**, 647–651 (1935).

and Vallarta, is not only fully capable of explaining theoretically all the experimental effects observed till now; but, in its present state of development, is in position to indicate the form and nature of future experiments which will serve to clarify definitely the problem of the exact composition of the charged components of cosmic radiation and its distribution in energy.

1. THE SYSTEM OF COÖRDINATES.

For the purpose of simplifying the discussion of the motion of a charged particle in the magnetic field of the earth it is convenient to modify the conventional system of spherical coördinates adopting, instead, the coördinates (ρ, λ, φ) whose polar axis OZ (Fig. 1) coincides with the negative direction

FIG. 1.



The adopted system of coördinates and the unit vectors i_ρ, i_λ, i_φ.

of the magnetic axis of the earth, where ρ denotes the distance from the geomagnetic center of the earth to the point in question, λ represents its magnetic latitude (regarded as positive in the northern hemisphere), and φ is the magnetic longitude considered as positive when measured towards the West.

The adopted system of spherical coördinates (Fig. 1) is characterized by means of three mutually orthogonal unit vectors i_ρ, i_λ, i_φ which, associated to a given point, denote the direction of the progressive tangents to the coördinate curves along which the three coördinates ρ, λ, φ increase respectively. It is observed that in a given point the unit vector i_ρ is vertical (towards the zenith), while the vectors i_λ, i_φ determine the horizontal plane with i_λ pointing towards the magnetic North of the earth and i_φ towards the West.

The cartesian coördinates (x, y, z) of a given point in the adopted inertial frame of reference (Fig. 1) are connected with the corresponding spherical coördinates by means of the relations:

$$\begin{aligned} x &= \rho \cos \lambda \sin \varphi, \\ y &= \rho \cos \lambda \cos \varphi, \\ z &= \rho \sin \lambda; \end{aligned} \tag{1}$$

from which it is readily shown that the components of the unit vectors i_ρ, i_λ, i_φ in such a reference system are given respectively by:

$$\begin{aligned} i_\rho &= (\cos \lambda \sin \varphi, \cos \lambda \cos \varphi, \sin \lambda), \\ i_\lambda &= (-\sin \lambda \sin \varphi, -\sin \lambda \cos \varphi, \cos \lambda), \\ i_\varphi &= (\cos \varphi, -\sin \varphi, 0). \end{aligned} \tag{2}$$

If we now assume that the spherical coördinates of a given point are analytic functions of a real parameter t, which we may later identify with the *time*; then, as t varies, the orientation of the three unit vectors i_ρ, i_λ, i_φ associated to the point in question will also vary. It becomes necessary, therefore, to compute the time derivatives of the unit vectors. This is readily attained from (2) by differentiating with respect to t the individual components, identifying then in the final results the components of the primitive vectors; thus:

$$\begin{aligned} (di_\rho/dt) &= (\dot{\lambda})i_\lambda + (\dot{\varphi} \cos \lambda)i_\varphi, \\ (di_\lambda/dt) &= (-\dot{\varphi} \sin \lambda)i_\varphi - (\dot{\lambda})i_\rho, \\ (di_\varphi/dt) &= (-\dot{\varphi} \cos \lambda)i_\rho + (\dot{\varphi} \sin \lambda)i_\lambda; \end{aligned} \tag{3}$$

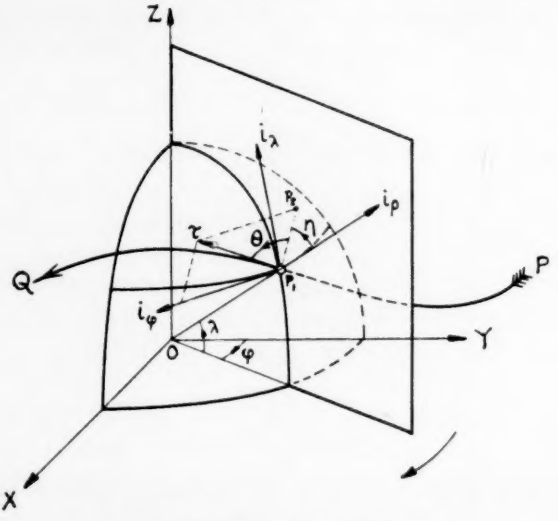
where the dots affecting various letters denote, as usual, the respective time derivatives.

2. KINEMATICS OF A MASS POINT IN THE ADOPTED SYSTEM OF COÖRDINATES.

The parametric representation of the trajectory of a moving particle whose spherical coördinates are ρ, λ, φ is obtained immediately by expressing these coördinates as functions of the independent variable t. The explicit knowledge of these functional relations, constituting in itself the solution of a given dynamical problem, is not essential to the study of the kinematics of a moving particle. In fact, it is sufficient to regard the position vector of a mass point, expressed in the adopted system of coördinates, as function of t and to compute, then, its first and second order derivatives; that is, if t denotes *time*, it is sufficient to compute the velocity and acceleration vectors of the particle.

In the adopted system of spherical coördinates the position vector of a point moving along a given trajectory PQ (Fig. 2)

FIG. 2.



The meridian plane and the angles θ and η.

becomes simply $(\rho)i_\rho$, where ρ denotes the distance OP_1 from the origin to the moving point. To compute the velocity vector

$$\bar{v} = (v_\rho)i_\rho + (v_\lambda)i_\lambda + (v_\varphi)i_\varphi, \qquad (4)$$

it suffices to take the first derivative with respect to time of the position vector; that is,

$$(d\bar{\rho}/dt) = (\dot{\rho})i_\rho + \rho(di_\rho/dt); \tag{5}$$

from which, making use of the first equation (3) to express (di_ρ/dt) in terms of the primitive vectors and identifying components by means of (4), the components of the velocity vector finally become:

$$v_\rho = \dot{\rho}; \qquad v_\lambda = \rho\dot{\lambda}; \qquad v_\varphi = \rho\dot{\varphi}\cos\lambda. \tag{6}$$

The acceleration vector of a moving particle,

$$\bar{a} = (a_\rho)i_\rho + (a_\lambda)i_\lambda + (a_\varphi)i_\varphi, \tag{7}$$

is likewise computed by taking the time derivative of the velocity vector (4), making use of equations (3) to express the derivatives of the unit vectors in terms of their components in the adopted frame of reference and, finally, utilizing (7) to identify the components of the acceleration vector:

$$\begin{aligned} a_\rho &= \ddot{\rho} - \rho\dot{\varphi}^2\cos^2\lambda - \rho\dot{\lambda}^2, \\ a_\lambda &= \rho\ddot{\lambda} + 2\dot{\rho}\dot{\lambda} + \rho\dot{\varphi}^2\sin\lambda\cos\lambda, \\ a_\varphi &= (1/\rho\cos\lambda)\frac{d}{dt}(\rho^2\dot{\varphi}\cos^2\lambda). \end{aligned} \tag{8}$$

The velocity vector of a moving particle coincides in direction with the progressive tangent along the trajectory, a fact easily shown by writing the velocity vector thus:

$$\bar{v} = \frac{d\bar{\rho}}{dt} = \frac{d\bar{\rho}}{ds}\frac{ds}{dt} = (v)\bar{\tau}, \tag{9}$$

where ds denotes the differential element of arc along the trajectory, $v = ds/dt$ represents the scalar magnitude of the velocity vector, and $\bar{\tau} = d\bar{\rho}/ds$ is the unit vector along the progressive tangent. Therefore, the components of the unit tangent vector are obtained from (6) by dividing the components of the velocity vector by its scalar magnitude; that is,

$$\bar{\tau} = (\dot{\rho}/v)i_\rho + (\rho\dot{\lambda}/v)i_\lambda + (\rho\dot{\varphi}/v)(\cos\lambda)i_\varphi. \tag{10}$$

3. THE FIELD OF FORCE OF A MAGNETIC DIPOLE.

The magnetic field of the earth may be represented, to a first approximation, by the field of a dipole of magnetic moment M, located at the origin of coördinates (Fig. 1), whose magnetic axis coincides with the negative direction of the polar axis OZ. The scalar potential of the magnetic dipole, in the adopted system of spherical coördinates, becomes

$$V = -\frac{M \sin \lambda}{\rho^2}, \tag{11}$$

where the absence of the φ-coördinate shows that the vector of magnetic intensity at a given point in space lies in the meridian plane containing the point in question, with its respective components along the coördinate directions in which ρ and λ increase being expressed by:

$$\begin{aligned} H_\rho &= -\frac{\partial V}{\partial \rho} = -\frac{2M \sin \lambda}{\rho^3}, \\ H_\lambda &= -\frac{1}{\rho}\frac{\partial V}{\partial \lambda} = \frac{M \cos \lambda}{\rho^3}. \end{aligned} \tag{12}$$

4. THE FORCE VECTOR ACTING ON A MOVING CHARGED PARTICLE.

If we now assume that a small particle of mass m and charge q (in electromagnetic units), possessing a vector velocity $\bar{v}$, comes under the action of the earth's magnetic field; then, the particle will describe at constant speed a more or less complicated trajectory, since the force vector

$$\bar{f} = (f_\rho)i_\rho + (f_\lambda)i_\lambda + (f_\varphi)i_\varphi \tag{13}$$

acting on the charged particle is proportional to the vector product of $\bar{v}$ and $\bar{H}$; that is,

$$\bar{f} = q(\bar{v} \times \bar{H}), \tag{14}$$

where $\bar{v}$ is the velocity vector of the charged particle (whose scalar magnitude is an invariant of the motion) and $\bar{H}$ is the magnetic intensity (12) in the field of the dipole. Since the force vector is always orthogonal to the trajectory described by the particle no work is done by the field and the kinetic energy of the particle is conserved throughout its motion.

To obtain the components of the force vector (13) it suffices to develop explicitly the vector product in (14) making use of equations (6) and (12) respectively to express the components of the vectors $\bar{v}$ and $\bar{H}$; thus:

$$\begin{aligned}(f_\rho/q) &= -v_\varphi H_\lambda = -(M\dot{\varphi}/\rho^2)\cos^2\lambda,\\ (f_\lambda/q) &= v_\varphi H_\rho = -(2M\dot{\varphi}/\rho^2)\sin\lambda\cos\lambda,\\ (f_\varphi/q) &= v_\rho H_\lambda - v_\lambda H_\rho = (2M\dot{\lambda}/\rho^2)\sin\lambda \\ &\qquad + (M\dot{\rho}/\rho^3)\cos\lambda.\end{aligned} \tag{15}$$

5. THE EQUATIONS OF MOTION OF A CHARGED PARTICLE.

The differential equations governing the motion of a charged particle in the field of a magnetic dipole are obtained at once by applying Newton's second law; that is, by equating the force vector (13) acting on the particle to the acceleration vector (7) multiplied by m, the relativistic mass of the particle. Since the scalar magnitude of the velocity vector along the trajectory of the particle is an invariant of the motion, the relativistic mass of the particle also remains constant with the result that the ordinary equations of classical mechanics are still applicable to the problem at hand. Expressing the results according to individual components the equations of motion become:

$$\begin{aligned}(m/Mq)(\ddot{\rho} - \rho\dot{\varphi}^2\cos^2\lambda - \rho\dot{\lambda}^2) &= -(\dot{\varphi}/\rho^2)\cos^2\lambda,\\ (m/Mq)(\rho\ddot{\lambda} + 2\dot{\rho}\dot{\lambda} + \rho\dot{\varphi}^2\sin\lambda\cos\lambda) &= -(2\dot{\varphi}/\rho^2)\sin\lambda\cos\lambda,\\ (m/Mq)(1/\rho\cos\lambda)\frac{d}{dt}(\rho^2\dot{\varphi}\cos^2\lambda) &= (2\dot{\lambda}/\rho^2)\sin\lambda + (\dot{\rho}/\rho^3)\cos\lambda;\end{aligned} \tag{16}$$

to which we must add the important relation denoting the conservation of kinetic energy:

$$\dot{\rho}^2 + \rho^2\dot{\lambda}^2 + \rho^2\dot{\varphi}^2\cos^2\lambda = v^2 \tag{17}$$

6. NORMALIZATION OF THE EQUATIONS OF MOTION.

The "Störmer" Unit of Length.

Before proceeding with a discussion of the equations of motion (16) and (17), particularly in what concerns the theory of cosmic radiation, it is convenient to introduce here a

change of variables, originally proposed by Störmer,[10] which has the remarkable property of normalizing the equations of motion; that is, of removing at once all the physical factors such as M, m, q, t, leaving only the metrical character of the motion.

The transformation due to Störmer consists of adopting, as unit of length, the measure

$$c_1 = \sqrt{M|q|/mv} \text{ cm}, \tag{18}$$

where $|q|$ denotes the absolute value of the electrostatic charge, M is the magnetic moment of the dipole in electromagnetic units, m is the relativistic mass of the particle and v the scalar magnitude of its velocity which, in accordance with earlier considerations, remains constant throughout the motion of the particle. Evidently, in the case of the motion of charged particles in the magnetic field of the earth, the *Störmer* unit of length depends only on the charge and on the kinetic energy of the particle, and, as such, may be regarded as a measure of the energy of particles of a given charge when used to measure a convenient standard length, for example, the radius of the earth.

To effect the normalization of the equations of motion it is sufficient to express the length of the radius vector and the differential element of arc $v\,dt$ along the trajectory in units of c_1 cm.; that is,

$$\begin{aligned} \rho &= c_1 r, \\ v\,dt &= c_1\,ds; \end{aligned} \tag{19}$$

where r and ds are pure numbers devoid of physical dimensions. Incorporating this transformation into the equations of motion one readily obtains the relations:

$$\begin{aligned} &\frac{d^2r}{ds^2} - r\left(\frac{d\lambda}{ds}\right)^2 - r\left(\frac{d\varphi}{ds}\right)^2 \cos^2\lambda = -\frac{\cos^2\lambda}{r^2}\frac{d\varphi}{ds}, \\ &r\frac{d^2\lambda}{ds^2} + 2\frac{dr}{ds}\frac{d\lambda}{ds} + r\left(\frac{d\varphi}{ds}\right)^2 \sin\lambda\cos\lambda \\ &\qquad = -\frac{2\sin\lambda\cos\lambda}{r^2}\frac{d\varphi}{ds}, \\ &(1/r\cos\lambda)\frac{d}{ds}\left(r^2\frac{d\varphi}{ds}\cos^2\lambda\right) = \frac{2\sin\lambda}{r^2}\frac{d\lambda}{ds} + \frac{\cos\lambda}{r^3}\frac{dr}{ds}; \end{aligned} \tag{20}$$

[10] Carl Störmer, *Pub. Univ. Obs. Oslo*, No. 10 (1934).

to which we must add, once more, the equation expressing the conservation of kinetic energy which, in units of c_1 cm., becomes:

$$\left(\frac{dr}{ds}\right)^2 + r^2\left(\frac{d\lambda}{ds}\right)^2 + r^2\cos^2\lambda\left(\frac{d\varphi}{ds}\right)^2 = 1. \qquad (21)$$

Störmer's transformation (19), in eliminating from the equations of motion all physical factors, reduces the problem of the motion of a charged particle in the field of a magnetic dipole to the solution of the normalized equations (20) and (21); for, it may be observed, that in order to study the motion of particles of any mass, charge, and velocity in the field of any magnetic dipole whatever, it is sufficient to agree to measure lengths not in cm. but in units of c_1 cm. in accordance with (19). The obvious advantages accruing from the use of this transformation, specially when dealing with the theory of the charged component of cosmic radiation, led Lemaître and Vallarta [11] to call the measure c_1 cm. the Störmer unit of length or, briefly, the *Störmer*, in honor of its inventor and discoverer.

7. FIRST INTEGRAL OF THE EQUATIONS OF MOTION. Störmer's Parameter γ_1.

The third of the equations of motion (20) may be written

$$\frac{d}{ds}\left(r^2\frac{d\varphi}{ds}\cos^2\lambda + \frac{\cos^2\lambda}{r}\right) = 0, \qquad (22)$$

which integrates at once yielding

$$r^2\frac{d\varphi}{ds}\cos^2\lambda + \frac{\cos^2\lambda}{r} = 2\gamma_1, \qquad (23)$$

where $2\gamma_1$ denotes the arbitrary constant of integration that may be readily evaluated by substituting in (23) the initial conditions of the problem at infinity. It is easily shown that the first term of (23) represents the component, along the positive direction of the magnetic axis of the dipole, of the vector characterizing the moment of momentum or angular momentum of the particle with respect to the origin of

[11] *Phys. Rev.*, **49**, 719–726 (1936).

coördinates. Therefore, the arbitrary constant of integration $2\gamma_1$, which is susceptible of assuming all values from $-\infty$ to $+\infty$, represents the projection on the magnetic axis of the dipole of the moment of momentum of the particle at infinity. Störmer's parameter γ_1 is to be regarded, then, as a constant of the motion of a given charged particle in the field of a magnetic dipole and, as such, it plays a dominant rôle in the theory of the charged component of a cosmic radiation.

8. THE EQUATIONS OF MOTION IN THE MERIDIAN PLANE.

Solving for $\frac{d\varphi}{ds}$ in the first integral of the motion (23) yields the expression:

$$\frac{d\varphi}{ds} = \frac{2\gamma_1}{r^2 \cos^2 \lambda} - \frac{1}{r^3}, \tag{24}$$

which makes it possible to eliminate the coördinate φ, by substitution, from the equations of motion (20) and (21); thus leading to the relations:

$$\begin{aligned} \frac{d^2r}{ds^2} - r\left(\frac{d\lambda}{ds}\right)^2 &= \frac{4\gamma_1^2}{r^3 \cos^2 \lambda} - \frac{6\gamma_1}{r^4} + \frac{2 \cos^2 \lambda}{r^5}, \\ r\frac{d^2\lambda}{ds^2} + 2\frac{dr}{ds}\frac{d\lambda}{ds} &= \frac{\sin \lambda \cos \lambda}{r^5} - \frac{4\gamma_1^2 \sin \lambda}{r^3 \cos^2 \lambda}, \\ \left(\frac{dr}{ds}\right)^2 + r^2\left(\frac{d\lambda}{ds}\right)^2 &= 1 - \frac{4\gamma_1^2}{r^2 \cos^2 \lambda} + \frac{4\gamma_1}{r^3} - \frac{\cos^2 \lambda}{r^4}; \end{aligned} \tag{25}$$

in which the last expression, relating to the kinetic energy of the particle, is not independent from the other two equations.

If we now imagine a meridian half-plane (Fig. 2) rotating around the polar axis OZ in such a way that the charged particle, throughout its complete trajectory, never abandons this plane; then, it is readily observed, the motion of a given particle in its own meridian plane is governed by the differential equations (25), the solution of which permits, by using (24), the study of the motion of the meridian plane. In the theory of the charged component of cosmic radiation the rotation of the meridian plane around the magnetic axis of the dipole does not play a fundamental rôle, except in the case

of the variation of total intensity which is attributed to the rotation of the solar galaxy.

9. ALLOWED AND FORBIDDEN REGIONS OF THE MERIDIAN PLANE.

Definition of the Angles θ and η.

To specify completely, at a given point of a trajectory, the direction of the progressive tangent (Fig. 2) it is sufficient to furnish the angle θ between the tangent to the trajectory and the meridian plane, and the angle η which the projection of the tangent on the meridian plane makes with the zenith direction. Evidently, from Fig. 2:

$$\sin \theta = \cos (\bar{\tau}, i_\varphi) = (\bar{\tau} \cdot i_\varphi) \tag{26}$$

which, expressing lengths in *Störmer* units and making use of equation (10), finally takes the form:

$$\sin \theta = r \cos \lambda \frac{d\varphi}{ds}. \tag{27}$$

Making use, once more, of the first integral of the motion in its form (23) to eliminate from (27) the coördinate φ, one readily obtains the important relation originally due to Störmer:[12]

$$\pm \sin \theta = 2\gamma_1/(r \cos \lambda) - (\cos \lambda)/r^2, \tag{28}$$

which is fundamental in the theory of cosmic radiation. In this expression the + sign refers to particles of positive charge and the − sign to particles of negative charge.

In the same manner, making use of the components of the unit vector along the progressive tangent (10), observing that the meridian plane is determined by the unit vectors i_ρ and i_λ, with the vector i_ρ along the zenith direction, and expressing lengths in *Störmer* units, one concludes at once that the angle η satisfies the expression:

$$\tan \eta = \frac{r\, d\lambda}{dr}. \tag{29}$$

The angle η is to be regarded as positive when measured towards the North regardless of the sign of the particle, whereas the angle θ is considered positive when measured

[12] *Pub. Univ. Obs. Oslo*, No. 10 (1934).

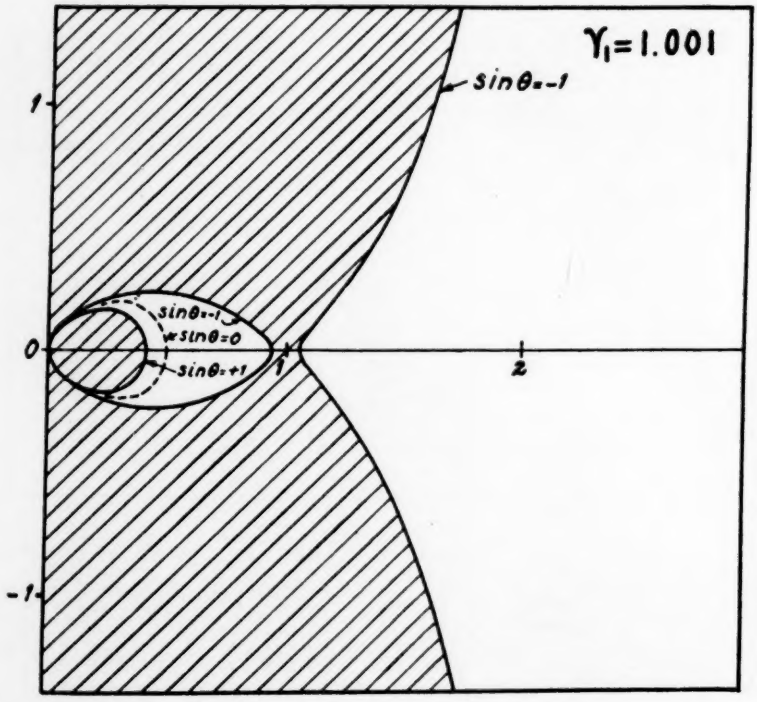
towards the East for positive particles or towards the West for negative particles. Therefore, in Fig. 2, assuming that the particle has a positive charge, θ is negative and η is positive.

The fundamental character of Störmer's formula (28) is readily brought out by observing that sin θ must satisfy the condition

$$-1 \leqq \sin\theta \leqq +1 \tag{30}$$

with the result that the equations of the boundaries between the allowed and forbidden regions of the meridian plane, in polar coördinates (r, λ), are obtained directly from (28) by placing respectively sin $\theta = +1$ and sin $\theta = -1$.

FIG. 3.

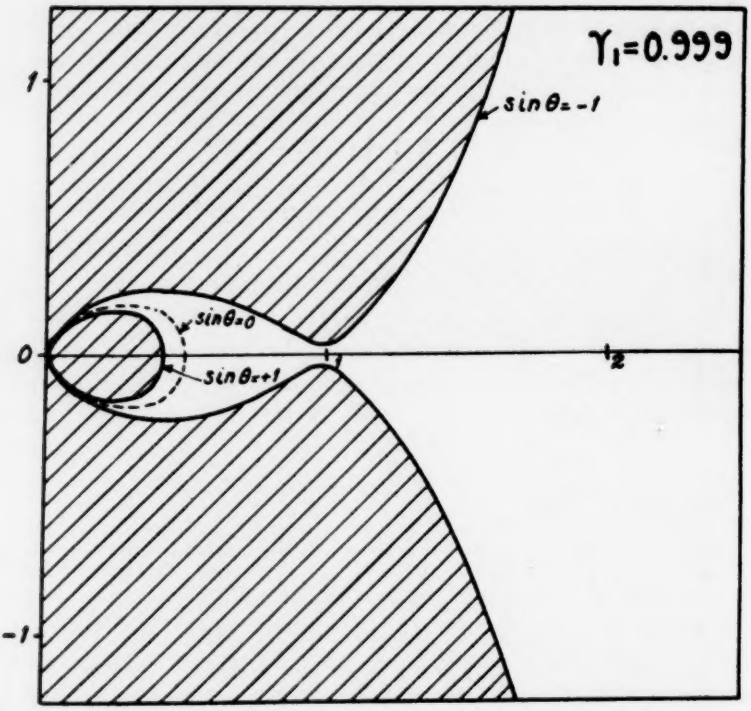


Figures 3, 4, 5, and 6, reproduced here from the works of Störmer,[13] illustrate the forbidden (cross-hatched) and allowed

[13] Ibid.

regions of the meridian plane corresponding to four values of the parameter γ_1. In these illustrations the intersection of earth's surface with the meridian plane (not shown) becomes a semi-circle whose radius, in *Störmer* units of length, depends naturally on the kinetic energy and nature of the particles

FIG. 4.

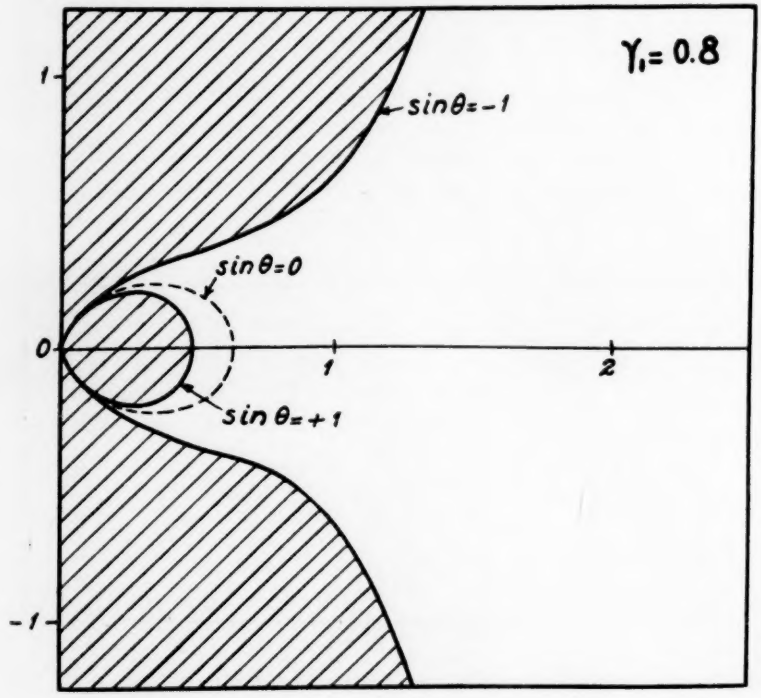


involved (Cf. ante Section 6). These figures show likewise the curve $\sin\theta = 0$, which is obtained from (28) and which represents, for a given value of γ_1, the locus of points at which the actual space trajectory of the particle is tangent to the meridian plane.

Figure 3, for $\gamma_1 > 1$, discloses the fact that the allowed region for energies $r < 1$ has no connection with the allowed region extending to infinity. Therefore, in the theory of cosmic radiation, the value $\gamma_1 = 1$ represents the upper

limit of Störmer's parameter, since particles of energies $r < 1$ originating at large distances from the earth are incapable of ever reaching any part of the terrestrial globe. For $\gamma_1 < 1$, Fig. 4 shows the appearance of the connecting pass between the two allowed regions and Figs. 5 and 6 exhibit

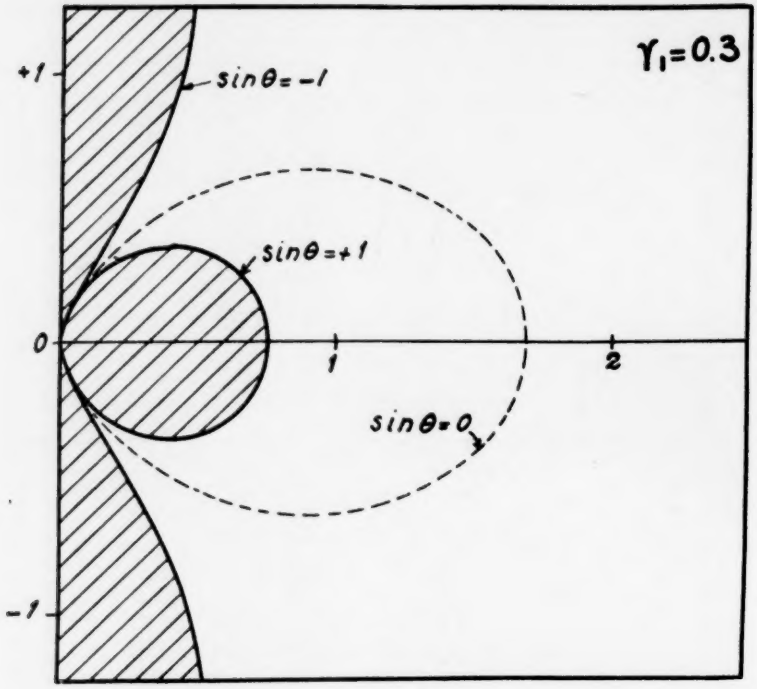
FIG. 5.



how this pass becomes wider as the value of γ_1 diminishes. It is evident from these illustrations that, for every value of $r < 1$ and for every latitude λ, there exist on the surface of the earth allowed and forbidden directions of cosmic rays. Lemaître and Vallarta have shown that for every point on the surface of the earth, and for particles of a given energy, the totality of allowed directions fills a cone of more or less irregular form, whose generators are the tangents to the trajectories which separate orbits coming from infinity from orbits arising in some other point of the terrestrial globe.

Among these trajectories, whose tangents form the boundary of the cone of allowed directions, the most important class corresponds to the trajectories asymptotic to the external

FIG. 6.



family of unstable periodic orbits originally discovered by Störmer.

10. APPLICATION OF LIOUVILLE'S THEOREM.

Following Lemaître's [14] presentation it is readily shown that within the cone of allowed directions the intensity of cosmic radiation arising from particles of a given energy is the same as the intensity at large distances from the earth where, it will be recalled, we have postulated the existence of a homogeneous and isotropic distribution of charged particles. This important conclusion, which can also be reached as a

[14] *Ann. de la Soc. Sci. de Bruxelles*, **A54**, 174–193 (1934).

corollary of Liouville's theorem,[15] is fundamental in the Lemaître-Vallarta theory of cosmic radiation; for, thanks to its consequences, there exists no problem of intensity in the theory of cosmic radiation, but simply the problem of determining the form of the cone of allowed directions for each energy and for each latitude.

Using cartesian coördinates, let

$$I(x, y, z, v_x, v_y, v_z)dx\,dy\,dz\,dv_x dv_y dv_z \tag{31}$$

represent the number of particles passing, at a given instant of time t, through a point (x, y, z) with a velocity v whose components are (v_x, v_y, v_z). A moment later, say at $t + dt$, these particles will be found in a neighboring domain $(x', y', z', v_x', v_y', v_z')$, their total number being

$$I(x', y', z', v_x', v_y', v_z')dx'dy'dz'dv_x'dv_y'dv_z'. \tag{32}$$

The necessary and sufficient condition for the intensity of the charged particles to remain constant along this bundle of trajectories (in phase-space) is expressed by demanding that the functional determinant or Jacobian

$$J = \frac{\partial(x', y', z', v_x', v_y', v_z')}{\partial(x, y, z, v_x, v_y, v_z)} = 1 \tag{33}$$

remain constant; that is, it is required to show that, in the expansion of J as a power series in dt, the coefficient of the first power vanishes. In fact, noting that

$$\begin{aligned} x' &= x + v_x dt + \cdots \\ &\cdots\cdots\cdots \\ v_x' &= v_x + a_x dt + \cdots \\ &\cdots\cdots\cdots \end{aligned} \tag{34}$$

where a_x, a_y, a_z denote the components of the acceleration vector, it is readily shown that the terms along the principal diagonal of the functional determinant J are respectively:

$$1, 1, 1, 1 + \frac{\partial a_x}{\partial v_x}dt, 1 + \frac{\partial a_y}{\partial v_y}dt, 1 + \frac{\partial a_z}{\partial v_z}dt; \tag{35}$$

[15] G. Lemaître and M. S. Vallarta, *Phys. Rev.*, **43**, 87–91 (1933).

and that the remaining terms of the determinant which do not vanish exhibit dt as a factor. Therefore, the desired power series expansion in dt begins as follows:

$$J = 1 + \left(\frac{\partial a_x}{\partial v_x} + \frac{\partial a_y}{\partial v_y} + \frac{\partial a_z}{\partial v_z} \right) dt + \cdots, \tag{36}$$

from which it is concluded that the intensity remains constant only when the expression within the parenthesis vanishes; that is, when the components of acceleration are independent of the components of velocity, a condition which is always satisfied in any electromagnetic field.

In particular, the case under consideration refers to the permanent field of a magnetic dipole in which the scalar magnitude of the velocity vector of a charged particle is an invariant of the motion. If we assume, furthermore, that the distribution of charged particles at large distances from the earth is homogeneous and isotropic, then, the distribution (31) may be characterized thus:

$$I(x, y, z, \omega)dx\, dy\, dz\, d\omega, \tag{37}$$

where $d\omega$ is the differential element of solid angle which comprises the directions of the trajectories under discussion and the appearance of which, in (37), arises from considering the number of particles of velocity ranging between v and $v + dv$; that is, by writing $v^2 dv d\omega$ instead of $dv_x dv_y dv_z$.

As a consequence of these developments it is observed that the intensity of cosmic radiation per unit solid angle, within the cone of allowed directions, for particles of a given charge and energy, is the same as the intensity at infinity, with the result that, in the theory of cosmic radiation, the fundamental problem becomes the evaluation of the total solid angle comprised, for every energy and for every latitude, within the cone of allowed directions.

11. THE CONFORMAL TRANSFORMATION OF THE MERIDIAN PLANE.

The equations of motion (25) in the meridian plane refer to the polar coördinates (r, λ) as dependent variables and to the arc-length s (in Störmers) along the trajectory as independent variable. To study these differential equations,

which cannot be integrated in general in terms of known functions, recourse must be had to methods of numerical [16] or mechanical integration.[17] For the purpose, Störmer [18] introduced the Goursat type conformal transformation of the meridian plane which is specified by means of the relations

$$r = \frac{1}{2\gamma_1} e^x,$$

$$ds = \frac{1}{(2\gamma_1)^3} e^{2x} d\sigma; \tag{38}$$

where x and σ are two new variables, dependent and independent respectively, which are to replace the variables r and s.

Figure 7 shows clearly the conformal nature of this transformation, for the circles r = constant and radial half-lines λ = constant of the $r - \lambda$ polar diagram become the straight lines x = constant and λ = constant of the cartesian diagram $x - \lambda$. It is readily seen from (29), on using (38), that

$$\tan \eta = \frac{d\lambda}{dx}, \tag{39}$$

which shows that the inclination of the tangent to a given trajectory in the $x - \lambda$ conformal map of the meridian plane corresponds, at a given point, to the angle η associated with the actual space trajectory. Furthermore, in the conformal map, although the rotation of the meridian plane about the polar axis has lost its physical significance, it is still possible to use Störmer's formula (28) for $\sin \theta$, which becomes in the new variables:

$$\pm \sin \theta = 4\gamma_1^2 e^{-x}(\sec \lambda - e^{-x} \cos \lambda), \tag{40}$$

and from which the locus of the points on the meridian plane corresponding to space trajectories which are tangent to the

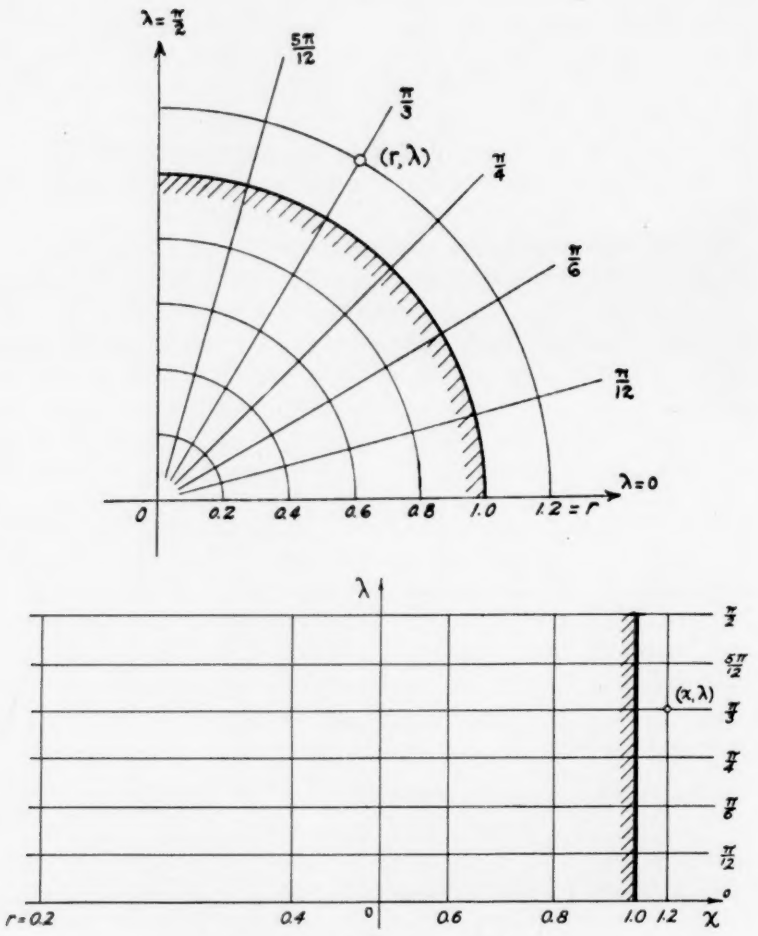
[16] Carl Störmer, *Astrophysica Norvegica*, **2**, No. 1, 1–121 (1936).
[17] G. Lemaître and M. S. Vallarta, *Phys. Rev.*, **49**, 719–726 (1936).
[18] *Zeits. f. Astrophys.*, **1**, 237–274 (1930).

meridian plane at such points ($\theta = 0$) is readily shown to be

$$x = 2 \ln \cos \lambda. \tag{41}$$

To carry out the transformation of the equations of motion

FIG. 7.



The conformal transformation of the meridian plane for $\gamma_1 = 1$:

$$r = \frac{1}{2\gamma_1} e^{x}; \qquad ds = \frac{1}{(2\gamma_1)^2} e^{+2x} d\sigma.$$

in the meridian plane it becomes necessary to compute

$$\begin{aligned}
\frac{dr}{ds} &= \frac{dr}{d\sigma}\frac{d\sigma}{ds} = (2\gamma_1)^2 e^{-x}\frac{dx}{d\sigma},\\
\frac{d\lambda}{ds} &= \frac{d\lambda}{d\sigma}\frac{d\sigma}{ds} = (2\gamma_1)^3 e^{-2x}\frac{d\lambda}{d\sigma},\\
\frac{d^2r}{ds^2} &= \frac{d\sigma}{ds}\frac{d}{d\sigma}\left(\frac{dr}{ds}\right) = (2\gamma_1)^5 e^{-3x}\left(\frac{d^2x}{d\sigma^2} - \left(\frac{dx}{d\sigma}\right)^2\right),\\
\frac{d^2\lambda}{ds^2} &= \frac{d\sigma}{ds}\frac{d}{d\sigma}\left(\frac{d\lambda}{ds}\right) = (2\gamma_1)^6 e^{-4x}\left(\frac{d^2\lambda}{d\sigma^2} - 2\frac{dx}{d\sigma}\frac{d\lambda}{d\sigma}\right).
\end{aligned} \tag{42}$$

Substituting these expressions into the equations of motion (25) and writing

$$a = 1/(2\gamma_1)^4 \tag{43}$$

for convenience, the desired transformation becomes:

$$\begin{aligned}
\frac{d^2x}{d\sigma^2} - \left(\frac{dx}{d\sigma}\right)^2 - \left(\frac{d\lambda}{d\sigma}\right)^2 &= \frac{1}{\cos^2\lambda} - 3e^{-x} + 2e^{-2x}\cos^2\lambda,\\
\frac{d^2\lambda}{d\sigma^2} &= e^{-2x}\sin\lambda\cos\lambda - \frac{\sin\lambda}{\cos^3\lambda},\\
\left(\frac{dx}{d\sigma}\right)^2 + \left(\frac{d\lambda}{d\sigma}\right)^2 &= ae^{2x} + 2e^{-x}\\
&\quad - e^{-2x}\cos^2\lambda - \frac{1}{\cos^2\lambda};
\end{aligned} \tag{44}$$

where, it will be recalled, only two of the three equations may be regarded as independent since the last equation in (44) arises from the equation dealing with the kinetic energy of the particle.

In terms of the independent parameter σ and the dependent variables x, λ of the conformal map, the final equations of motion are obtained by adding the first and third equations of (44) keeping the second equation intact. In short, writing

$$P = \left(\frac{dx}{d\sigma}\right)^2 + \left(\frac{d\lambda}{d\sigma}\right)^2 = ae^{2x} - (e^{-x}\cos\lambda - 1/\cos\lambda)^2; \tag{45}$$

then, the equations of motion become

$$\frac{d^2x}{d\sigma^2} = \frac{1}{2}\frac{\partial P}{\partial x} = ae^{2x} - e^{-x} + e^{-2x}\cos^2\lambda,$$

$$\frac{d^2\lambda}{d\sigma^2} = \frac{1}{2}\frac{\partial P}{\partial \lambda} = e^{-2x}\sin\lambda\cos\lambda - \frac{\sin\lambda}{\cos^3\lambda}.$$

The solution of these equations, by mechanical and analytical methods, both in the case of the family of periodic orbits which this system of equations admits in the proximity of the pass and in the case of asymptotic trajectories which constitute, at a given point, generators of the allowed cone of cosmic radiation, represents essentially the work of Lemaître and Vallarta who have published in the last six years more than ten papers, each one representing the elimination of a new obstacle and a decisive step towards the complete solution of the fundamental problem in the theory of cosmic radiation; namely, the determination of the composition of cosmic radiation and its distribution in energy.

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Note: The preceding list contains only those publications by Störmer which concern in some way the theory of cosmic radiation. In particular, publications (15) to (20) inclusive constitute the six communications that have appeared under the title: "The Trajectories of Electric Particles in the Field of a Magnetic Dipole with Application to the Theory of Cosmic Radiation."

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Education.—L. MIES VAN DER ROHE of Armour Institute, Chicago, said in a recent address, "Any education must be directed, first of all, towards the practical side of life. But if one may speak of real education, then it must go further and reach the personal sphere and lead to a moulding of the human being. The first aim should be to qualify the person to maintain himself in every day life. It is to equip him with the necessary knowledge and ability for this purpose. The second aim is directed towards a formation of the personality. It should qualify him to make the right use of his knowledge and ability. Genuine education is aimed not only towards specific ends but also towards an appreciation of values. Our aims are bound up with the special structure of our epoch. Values, on the contrary, are anchored in spiritual destination of mankind. The ends, towards which we strive, determine the character of our civilization, while the values we set determine our cultural level. Although aspirations and values are of different nature and of different origin, they are actually closely associated. For our standards of value are related to our aspirations, and our aspirations obtain their meaning from these values. Both of these concepts are necessary to establish full human existence. The one assures the person his vital existence, but it is only the other that makes his spiritual existence possible. Just as these propositions have a validity for all human conduct, even for the slightest differentiation of value, so are they that much more binding in the realm of architecture. Architecture is rooted with its simplest forms entirely in the useful, but it extends over all the degrees of value into the highest sphere of spiritual existence, into the sphere of the significant; the realm of pure art."

R. H. O.

GRAPHICAL ANALYSIS OF FREE VIBRATIONS OF HELICAL SPRINGS.*

BY

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Helical springs are used as measuring elements for variable forces in several kinds of engineering instruments such as engine indicators and dynamometers. Their vibration characteristics have an important influence on the behavior and accuracy of such instruments. Therefore the following problem is of theoretical and practical importance:

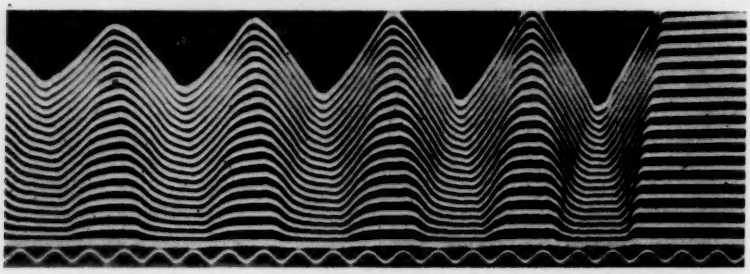
Consider a helical spring, with or without concentrated masses attached to its two ends, initially at rest and put under an initial longitudinal strain by means of external constraining forces. Determine the law of motion if the constraining forces are instantaneously removed.

The term "helical spring" includes also elastic rods. In a broader sense the above problem applies also to helical springs or elastic rods in torsion by substituting "moment of inertia" instead of "mass," "tangential strain" instead of "longitudinal strain" and "couples" instead of "forces." This problem has been treated by several authors (Krylow, Holm, Thamm, Sayre, Ref. 10, 11, 4, 12) under the assumption that the strain at all points of the spring is the same at any given instant of time, which assumption implies an infinite velocity of propagation. The mass of the spring was taken into consideration by either adding a certain calculated fraction of the spring mass to the concentrated mass or by multiplying with a calculated factor the period calculated for the concentrated masses only. Owing to these simplifying assumptions the analytical method is not fully satisfying. It is inaccurate because it yields a purely harmonic motion, and it is incomplete because the law of motion for the intermediate points of the spring is not supplied.

* Received July, 1938.

Actually the velocity of propagation of a disturbance in a spring is a finite quantity and therefore the strain in a spring element changes not only from instant to instant, but also from point to point in the spring. As a result the motion will differ from the pure harmonic as the beautiful photographs of W. Weibull show (Fig. 1). This fact has been taken into

FIG. 1.



Photograph of free vibrations, as a function of time, of an unloaded spring, by W. Weibull.

consideration (Weibull, Ref. 2; Magg, Ref. 1) but the calculations it involves are impracticably complicated and have been carried out only for some simple particular cases.

The graphical method used by the author in several previous publications (Ref. 5–9) avoids these difficulties and is easy to perform. For a complete description of the phenomenon the (1) velocity v, (2) strain s, and (3) displacement u has to be given for all points x of the spring and for all instants of time. Expressed in analytical terms the following three functions

$$v = f_1(t, x)$$

$$s = f_2(t, x)$$

$$u = f_3(t, x)$$

are sought. In graphical terms each of these functions can be represented by a tri-dimensional "stereogram," with the v, s and u values as ordinates erected over the t–x plane as base. The problem then resolves itself into finding the data from which these three stereograms can be constructed.

This is possible with the aid of the following relationships:

(*a*) The location of the disturbance front can be represented in the t–x diagram by straight lines having the slope tan ($\pm \varphi$), i.e.:

$$\frac{\text{progress of disturbance}}{\text{in a given time element}} = \frac{\Delta x}{\Delta t} = \pm a = \tan (\pm \varphi),$$

a being the velocity of propagation of the disturbance; these lines will be referred to as "disturbance lines."

(*b*) The velocity-change is proportional to the corresponding strain-change and can be represented in the v–s diagram by straight lines having the slope tan ($\pm \alpha$), i.e.:

$$\frac{\text{change of strain}}{\text{change of velocity}} = \frac{\Delta s}{\Delta v} = \frac{\text{strain corresponding to stress } E}{\text{velocity of propagation}}$$

$$= \frac{1}{a} = \tan (\pm \alpha),$$

which will be referred to as "directrices."

(*c*) The displacement u can be determined from either the v or s stereogram with the aid of the following relationships:

$$\frac{du}{dt} = v \qquad \text{and} \qquad \frac{du}{dx} = Ls,$$

hence

$$u = \int v dt \qquad \text{and} \qquad u = L \int s dx,$$

i.e., the displacement can be determined by integrating either the v or s stereograms.

The derivations of (*a*) and (*b*) are given in Ref. 6 and 9 and can be omitted here.

The method will be now applied to a number of assumed conditions having special interest.

I. SPRING WITHOUT CONCENTRATED MASS.

(a) One End Fixed, the Other End Free (Fig. 2).

It is assumed that the spring with the unstressed length L_0 is compressed to the length L_1, involving an initial strain s_1 and initial velocity $v_1 = 0$, as characterized by point 1 in

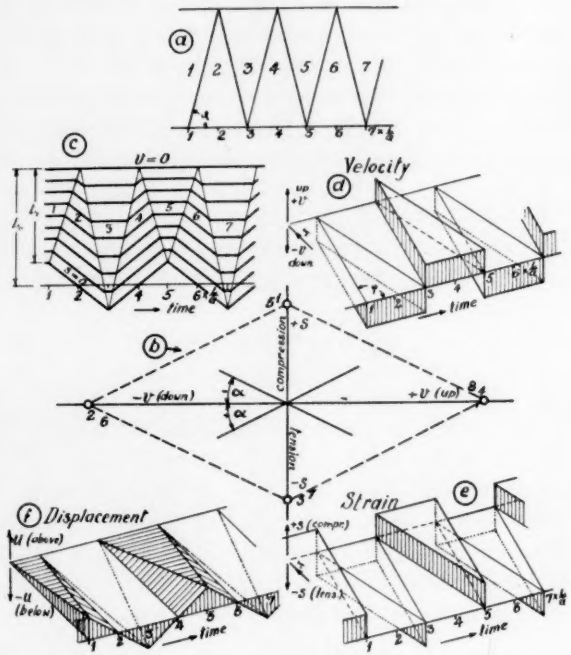
the v–s diagram. At the instant 1 the unattached end of the spring is instantaneously released, whereby the strain at that end vanishes ($s_2 = 0$) and the velocity assumes the value of v_2, the new state being defined by point 2 situated on the directrix drawn from point 1 in the v–s diagram, Fig. 2b. The change of state constitutes a disturbance which proceeds along the spring with the velocity of propagation a and after the elapse of L/a time arrives at the fixed end. Here the velocity is reduced to 0 and the strain correspondingly changes to s_3 defined by point 3 situated on the directrix drawn from point 2 in the v–s diagram. The change from state 2 to state 3 returns towards the free end of the spring with the velocity of propagation and arrives there after the elapse of another L/a time. Here its strain is reduced to zero and its velocity assumes v_4 as defined by point 4 in the v–s diagram. Continuing this construction the state of the spring according to points 5, 6, 7, 8 and so on can be determined, each state being valid for the triangular territories marked with the same numbers in the t–x diagram, Fig. 2a.

The change of position of the coils, as a function of time, is illustrated on Fig. 2c. The slope of the heavy lines is proportional to the velocity; the relative change of distance between them, as measured on any t = const. ordinate, is proportional to the strain. The disturbance lines appear as the sloping lines connecting the two ends of the spring taken at L/a time interval apart.

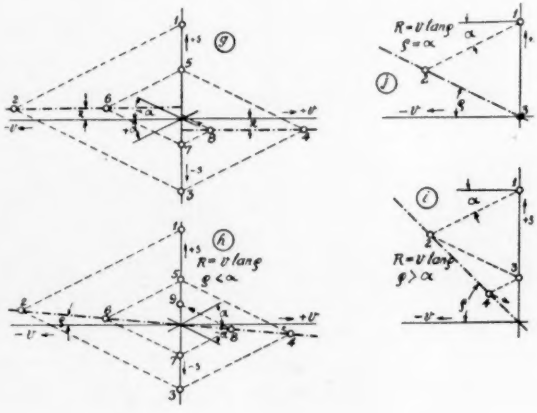
Erecting the v and s coordinates (in the v–s diagram) of the 1, 2, 3, 4 $\cdots$ points over the t–x plane as base the velocity- and strain-stereograms can be built, which are shown in parallel perspective in Figs. 2d and 2e. The displacements are obtained by forming the $u = \int vdt$ lines or the $u = L\int sdx$ lines over the t–x plane as base, as shown by the u-stereogram, Fig. 2f. These stereograms define the motion of the spring for every point in the spring and for every instant of time. From these the following characteristics of the vibration can be read:

(1) The velocity at the fixed end is 0, and the strain at the free end is 0 at all times.

FIG. 2.



Unloaded spring fixed at one end, free on the other end. (a) time-disturbance diagram; (b) velocity-strain diagram; (c) displacement vs. time; (d) velocity-stereogram; (e) strain-stereogram; (f) displacement-stereogram; (g) effect of Coulomb friction; (h) effect of periodic damping; (i) aperiodic damping; (j) critical damping.



(2) The motion at the free end is periodic with a period $T = 4L/a$; it is however not harmonic, but has constant positive and negative velocities alternating after the elapse of each consecutive $2L/a$ interval (cf. Appendix II).

(3) The motion of the intermediate points of the spring is intermittent, the durations of positive and negative velocities being separated by dwells of zero velocity.

Friction and Damping

Friction (Coulomb type) is equivalent to a definite, constant value of strain R which must be exceeded before motion, according to

$$\Delta v = (\Delta s - R) \tan \alpha,$$

can result. In the graphical construction friction is represented by two lines in the v–s diagram, parallel to the v-axis drawn at $s = + R$ and $s = - R$ distance from it (Fig. 2g). It is seen that the velocity and strain alternate between positive and negative values but the absolute magnitude of their consecutive values is not constant but decreases in an arithmetic progression. The motion stops altogether when the strain drops below the value of friction, i.e.,

$$s < |R|.$$

Damping (or molecular friction) is equivalent to a strain R which is not constant but proportional to the velocity, i.e.,

$$R = v \tan \rho,$$

where $\tan \rho$ is the proportionality factor. This value of strain must be exceeded before motion can result, and only the excess of strain can be converted into velocity according to the equation

$$\Delta v = (\Delta s - R) \tan \alpha = (\Delta s - v \tan \rho) \tan \alpha.$$

In the graphical construction damping is represented by a straight line passing through the point of origin of the v–s diagram forming an angle ρ therewith (Fig. 2h). It is seen that the consecutive absolute values of velocity and of strain decrease in geometric progression. If $\rho < \alpha$ the damping is periodic and the motion ceases only after the elapse of an infinite number of L/a intervals, vibrating on both sides of its

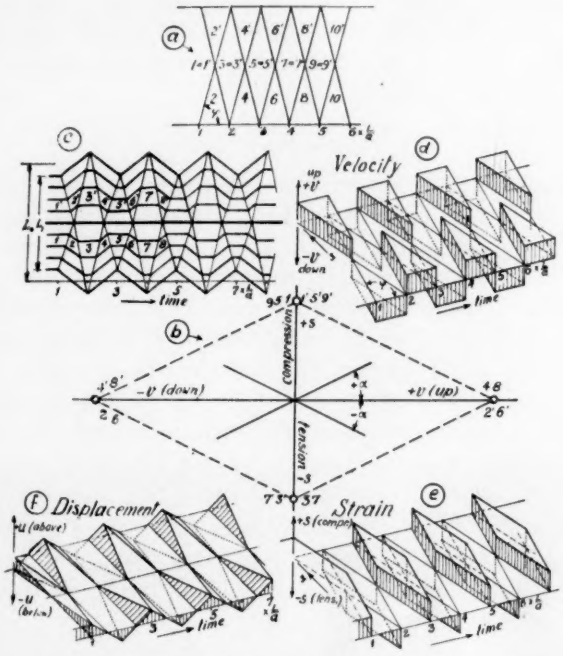
position of equilibrium with ever-decreasing amplitude. If $\rho < \alpha$ (Fig. 2i) the damping is aperiodic and no overshooting the equilibrium position takes place. In the case of critical damping $\rho = \alpha$ (Fig. 2j) the whole potential energy of the spring is consumed by the damping in $2L/a$ period after which the spring comes to a standstill in its position of equilibrium.

Other varieties of damping, e.g., in which the damping force is not a linear function of the velocity, but that of a higher power of it, or some other not necessarily algebraic function of the velocity, can be represented in the v–s diagram and the graphical construction performed in the manner shown.

(b) Both Ends Free (Fig. 3).

It is assumed that the spring with the unstressed length L_0 is compressed to L_1 length, the initial state, $v_1 = 0$, $s = s_1$

FIG. 3.



Unloaded spring free on both ends. (a) time-disturbance diagram; (b) velocity-strain diagram; (c) displacement vs. time; (d), (e), (f) stereograms of velocity, strain and displacement.

being defined by the point 1 in the v–s diagram, Fig. 3b. At the instant 1 the spring is instantaneously released. The strain at the ends vanishes and the upper end assumes a positive (upward) velocity, the lower end assumes a negative (downward) velocity, as defined by the points $2'$ and 2 in the v–s diagram. Thus simultaneously two disturbances are initiated, one from each end of the spring, traveling in opposite directions. After the elapse of $(1/2)(L/a)$ time these meet at the mid-point of the spring and the new state $3 = 3'$ results. The successive states 4, 6, 8 for the upper free end, $4'$, $6'$, $8'$ for the lower free end and $5 \equiv 5', 7 \equiv 7' \cdots$ for the mid-point of the spring can be found from the v–s diagram (Fig. 3b), and the t–x zones for which the various states are valid can be found in the triangles and rhomboids formed by the disturbance lines in the t–x diagram (Fig. 3a). The stereograms of velocity, strain and displacement constructed from the data thus obtained are shown in Figs. 3d, e, and f. It is seen that the mid-point is stationary and assumes alternating s_1 and s_3 stresses, while the two ends have zero stress and assume alternating v_2 and v_4 velocities, the period of motion being $T = 2L/a$ (cf. Appendix II).

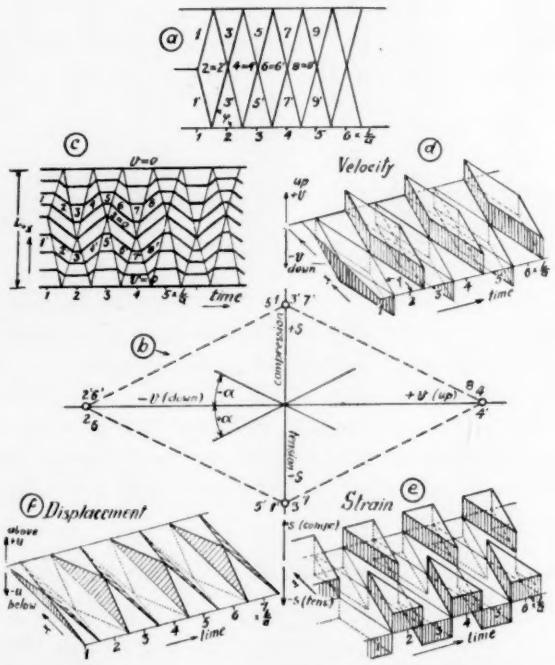
(c) Both Ends Fixed (Fig. 4).

It is assumed that the ends of the spring are fixed at a distance equal to the unstressed length L_0 of the spring. The mid-point is displaced upward, causing in the upper half a compressional strain corresponding to point 1 in the v–s diagram, and in the lower half a tensional strain corresponding to point $1'$. At the instant 1 the mid-point is instantaneously released. The strain at the mid-point vanishes and it assumes a velocity corresponding to point $2 = 2'$ in the v–s diagram which velocity spreads toward the two ends with the velocity of propagation. On reaching, after the elapse of $L/2a$ time, the two fixed ends the velocity is reduced to 0 and at the lower end a compressional strain according to point $3'$, and at the upper end a tensional strain according to point 3 results. The zones having the same v–s values are delimited by the disturbance lines as shown in Fig. 4a.

Figures 4d, e and f are the stereograms of velocity, of strain and of displacement respectively which give full information

on the motion of the spring. From these it can be seen that the strain of the mid-point is constant (= 0) at all times, and its velocity alternates between the two values v_2 and v_4, the period of motion being $T = 2L/a$ (cf. Appendix II). The velocity of the two ends is zero at all times and the strain alternates between the values s_1 (compression) and s_3 (tension). The points intermediate between the mid-point and

FIG. 4.



Unloaded spring fixed at both ends. (a) time-disturbance diagram; (b) velocity-strain diagram; (c) displacement vs. time; (d), (e), (f) stereograms of velocity, strain and displacement.

the ends execute an intermittent motion, the durations of positive and negative velocities being separated by dwells of zero velocity.

It is interesting to compare Fig. 3 with Fig. 4. The velocity-stereogram Fig. 3d is identical with the strain stereogram Fig. 4e, and Fig. 3e with Fig. 4d. The stereogram of displacement Fig. 3f is an equal configuration with Fig. 4f turned by 90 degrees, i.e., with the x- and t-axes interchanged.

II. SPRINGS WITH CONCENTRATED MASSES.

If a spring force sEF is applied to a mass M during a time-duration Δt a velocity change Δv is imparted to the mass according to the law of momentum:

$$sEF\Delta t = M\Delta v,$$

whence

$$\frac{s}{\Delta v} = \frac{M}{EF}\frac{1}{\Delta t},$$

choosing $\Delta t = L/a$, i.e., the duration of a single traversal of a disturbance in the spring, the following formula is obtained:

$$\frac{s}{\Delta v} = \frac{M}{EF}\frac{a}{L}.$$

Substituting: $E = a^2\rho$; $LF\rho = m$ we obtain

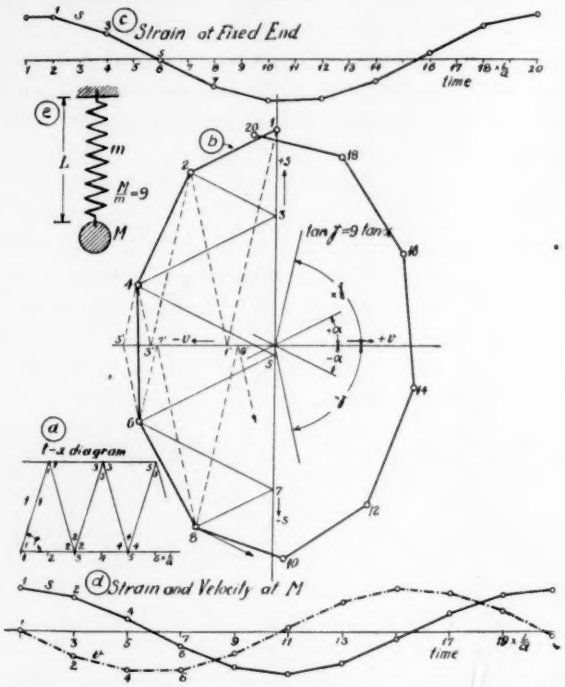
$$\frac{s}{\Delta v} = \frac{M}{m}\frac{1}{a} = \tan\gamma = \frac{M}{m}\tan\alpha = \frac{\sigma}{a}$$

as expressing the interrelationship of velocity change undergone by the concentrated mass when acted upon by the spring force corresponding to the strain s during a single interval L/a. In the v–s diagram this velocity change can be represented by a straight line having the slope M/m times greater than the $\Delta s/\Delta v$ directrix. This result will be now applied to a spring having

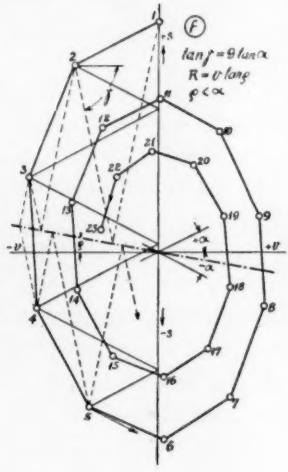
(d) One End Fixed, the Other End Attached to a Mass (Figs. 5 and 6).

It is assumed that the spring is compressed to a strain s_1 and kept at rest $v_1 = 0$ as defined by point 1 in the v–s diagram, Fig. 5b. At the instant 1 the spring is instantaneously released. As a result the velocity of the free-end, and of the mass attached to it, increases and the stress decreases along the 1–2 directrix and this change spreads towards the fixed end with the velocity of propagation a. During the first L/a period the velocity of the free end and of the mass will attain the value v_1 as defined by the intersection 1′ of the v axis with the tan γ line. During the second L/a interval a further increase of velocity takes place according to the tan $(-\gamma)$ line and at the end of the $2L/a$ interval the velocity will be v_2 as

FIG. 5.

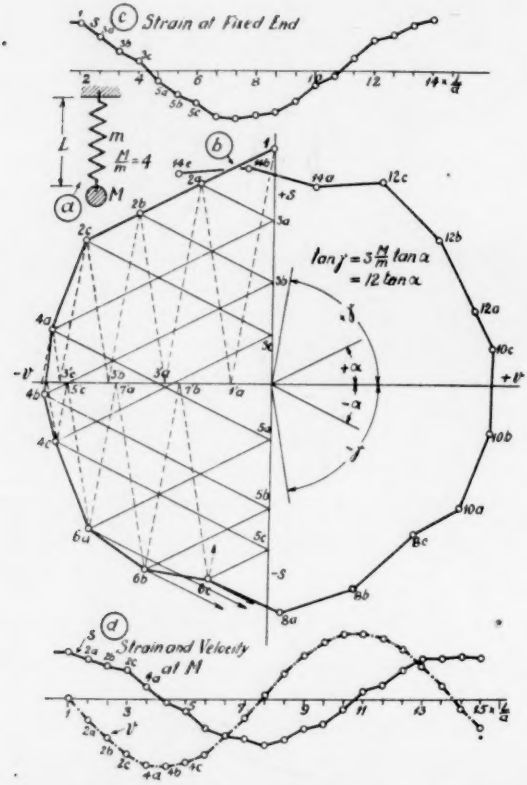


Spring fixed at one end, loaded with concentrated mass on the other end. The time interval during which the strain is assumed to be constant is L/a. (a) time-disturbance diagram; (b) v–s diagram; (c) strain at fixed end vs. time; (d) strain and velocity at movable end vs. time; (e) schematic arrangement; (f) effect of periodic damping.



defined by the point 2 situated on the intersection of the tan α directrix drawn from the 1 point, with the tan $(-\gamma)$ directrix drawn from the 1′ point. This construction is based on the assumption that during the first L/a interval the force corresponding to the constant s_1 strain, during the second interval

FIG. 6.



Spring fixed at one end, loaded with concentrated mass on the other end. The time-interval during which the strain is assumed to be constant is $(1/3)(L/a)$. (a) schematic arrangement; (b) v–s diagram; (c) and (d) strain and velocity history at the two ends.

the force corresponding to the constant s_2 strain acted upon the mass. This is not strictly correct, because the strain has been continuously changing during this $2L/a$ interval, but the approximation is close, because s_1 is greater than the average strain, and s_2 is less than the average strain, and the two errors

fairly well cancel out (see Appendix III). In a similar manner the motion at the end of the second $2L/a$ interval will be defined by the intersection of the directrix tan α drawn from the $3'$ point. Continuing this construction the consecutive 3, 4, 5 $\cdots$ points are obtained, which, as it is evident from the v–s diagram, lie on a polygon resembling an ellipse. The complete stereograms could be easily drawn, but are omitted here, and only the $s = f(t)$ line for the fixed end (Fig. 5c) and the $s = f(t)$ and $v = f(t)$ lines for the movable end (Fig. 5d) are shown. These show an approximately, but not exactly, harmonic interrelationship, the period of vibration being about 19.5 L/a, showing a good agreement with the value 19.3 L/a calculated from the formula, Appendix III. The deviation is due partly to our assumption of constant instead of the actually varying strains during each L/a interval, and partly to the fact that the actual motion being not truly harmonic the "period" is not strictly constant but varies slightly from cycle to cycle.

The influence of friction and damping can be taken into consideration in a manner similar to that treated in Figs. 2g–j. In Fig. 5e the v–s diagram is shown assuming the presence of periodic damping as represented by the tan ρ characteristic line passing through the point of origin, $\rho < \gamma$. It is seen that the consecutive peak values of velocity and strain decrease in an apparently geometric progression, resulting in a "spiral polygon"-shaped v–s diagram. Aperiodic ($\rho > \gamma$) and critical ($\rho = \gamma$) damping can be treated in a similar manner.

The assumption of constant strain was allowable in this case, because the $M/m = 9$ ratio was fairly high, and therefore the change of strain during an L/a interval not great. If the M/m ratio is small then this assumption is no longer permissible. In such cases a submultiple of L/a has to be used for the determination of the tan γ directrix. This more accurate construction is illustrated in Fig. 6 in which an $M/m = 4$ ratio is assumed, and the velocity change of the mass is determined for an $(1/3)$ (L/a) duration;

$$\frac{s}{\Delta v} = \frac{1}{3}\frac{M}{m}\tan \alpha.$$

Using this directrix for each $2L/a$ period three points are obtained instead of the one point in the previous example. The construction is evident from the drawing. The strain for the fixed end, and the strain and velocity for the movable end, are shown in Figs. 6c and d on a time basis. The period of motion is approximately 13.1 L/a which agrees with the 13.1 L/a value obtainable by calculation (Appendix III).

It is seen that the first side 1–2 of the polygon, representing the change of velocity and strain during the first $2L/a$ interval is truly a straight line. The other sides of the polygon are straight lines only in the first approximation (Fig. 5b), become themselves broken lines in a closer approximation (Fig. 6b), and strictly speaking would become curved lines as the Δt interval (during which the strain producing the velocity change is assumed constant) is made to approach zero. The polygons obtained during the second, third and following circumambulations are not the same but differ from each other. It is reasonable to suspect that ultimately the polygon would lose its corners and irregularities and develop into a true ellipse as the stationary condition is attained.

(e) Both Ends Free and Attached to Masses (Fig. 7).

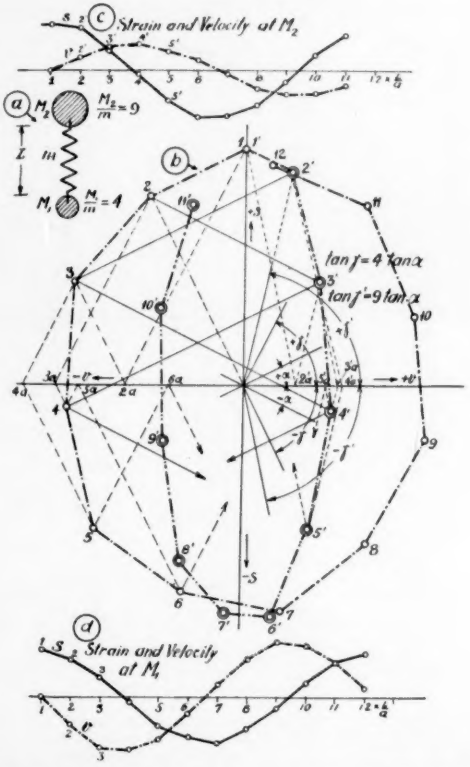
It is assumed that the spring is compressed to an initial strain s_1 (point 1) and at the instant 1 instantaneously released. The velocity change of M_1 will take place according to the $M_1/m \tan \alpha = \tan \gamma$ directrix, that of M_2 according to the $M_2/m \tan \alpha = \tan \gamma'$ directrix. The change of strain spreads from M_1 to M_2 and from M_2 to M_1, and can be represented in the v–p diagram by the points of intersection of the tan α and tan γ directrices on the one hand, and of the tan α and tan γ' directrices on the other hand. The motion of the two masses is defined by the two ellipse-like polygons, the larger mass having the smaller maximum velocity and the smaller mass having the larger maximum velocity in inverse ratio to the respective masses. The strain and velocity histories are shown in Figs. 7c and d for both masses. The period of vibration is approximately 11 L/a which agrees well with the value 10.7 L/a obtainable by calculation, Appendix III. At the cost of more labor a more accurate determination

would be possible by using a submultiple of the L/a interval, e.g.,

$$\tan \gamma_1 = \frac{1}{3}\frac{M_1}{m}\tan \alpha \qquad \text{and} \qquad \tan \gamma_2 = \frac{1}{3}\frac{M_2}{m}\tan \alpha$$

as was done in the previous example.

FIG. 7.



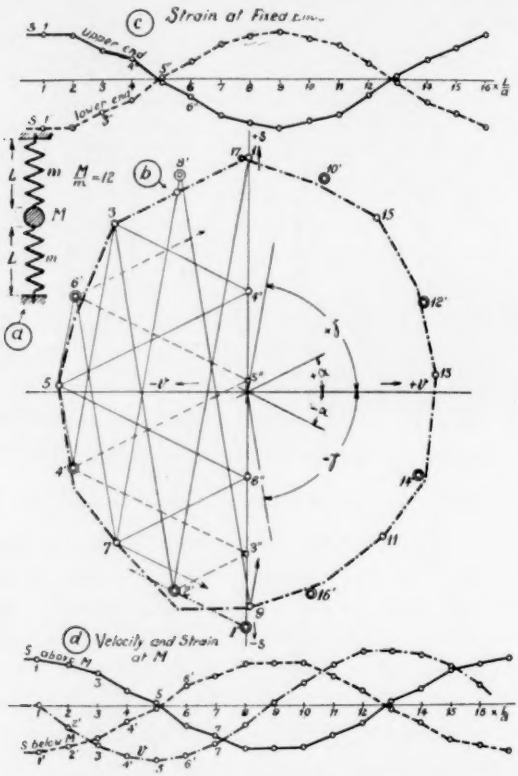
Spring loaded with finite concentrated masses at both ends. The time-interval during which the strain is assumed to be constant is L/a. (a) schematic arrangement; (b) v–s diagram; (c) and (d) strain and velocity history at the two ends.

(f) Both Ends Fixed, a Finite Mass Attached to the Mid-point (Fig. 8).

It is assumed that the two ends of the spring are fixed at the unstressed length L_0, a mass M attached to the mid-point, and displaced upward, causing an initial compressional

strain s_1 (point 1) in the upper half of the spring, and an equal tensional strain s_1' (point 1′) in the lower half of the spring. At the instant 1 the mid-point together with the mass M is instantaneously released. The mass will assume a downward

FIG. 8.



Spring fixed at both ends, loaded with a concentrated mass at the mid-point. (a) schématic arrangement; (b) v–s diagram; (c) and (d) strain and velocity history at the two ends.

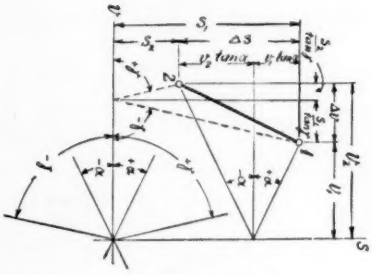
directed velocity, and the strain at the two sides of it will decrease, according to the 1–2 and 1′–2′ directrices respectively. The increase of the velocity is defined by the directrix tan γ drawn from the point 1 and brought into intersection with the directrix tan α passing through the 1′ point. (Same result is obtained if the role of the 1 and 1′ points is interchanged.) The construction is clearly visible from the dia-

gram. The history of strain for the two ends, and the history of velocity for the mass is also shown in Figs. 8c and d. The period of vibration is about 15.7 L/a intervals which agrees well with the calculated value $T = 15.8\ L/a$ (Appendix III).

As a closing example, proving also the validity of this graphical method, the

(*f*) *Natural period of mass suspended on a massless spring* will be derived on the basis of the graphical procedure. In Fig. 9 the graphical construction of two subsequent stages 1

FIG. 9.



Velocity-stress diagram for the calculation of motion of a finite mass on a massless spring.

and 2 is shown from which it can be written:

$$\Delta v = (s_1 + s_2)\frac{1}{\tan \gamma} = (s_1 + s_2)a\frac{m}{M}$$

and

$$\Delta s = -(v_1 + v_2)\tan \alpha = -(v_1 + v_2)\frac{1}{a}.$$

Denoting the displacement of the spring end by u the following relationships hold: $\Delta u = L\Delta s$; $u = Ls$,

$$\frac{\Delta u}{\Delta t} = v,$$

$$\frac{\Delta u}{\Delta v} = -\frac{v_1 + v_2}{u_1 + u_2}\frac{L^2}{a^2}\frac{M}{m}.$$

If, now, the mass m of the spring is made to approach 0, it is equivalent to the velocity of propagation a approaching infinity; $\Delta u/\Delta v$ will approach du/dv; $v_1 + v_2\ (=)\ 2v$ and

$u_1 + u_2 (=) 2u$, and the differential equation of motion can be written:

$$\frac{du}{dv} = -\frac{v}{u}\frac{M}{m}\left(\frac{L}{a}\right)^2,$$

the solution of which is:

$$u^2 + v^2\frac{M}{m}\left(\frac{L}{a}\right)^2 - u_0^2 = 0.$$

This is the equation of an ellipse, showing that the displacement and the velocity are harmonic functions of time, the frequency being:

$$\omega = \sqrt{\frac{1}{\frac{M}{m}\left(\frac{L}{a}\right)^2}},$$

from which the natural period

$$T = \frac{2}{\omega} = 2\pi\sqrt{\frac{M}{m}}\frac{L}{a} = 2\pi\sqrt{\frac{M}{c}}\text{ g.e.d.}$$

The foregoing examples show that the graphical method is easy to apply and once its simple principles have been mastered it can be carried out almost mechanically, resembling in this respect the procedures of graphic statics. It facilitates forming a clear mental picture of the phenomena and its results show a satisfactory agreement with the results of calculation. Its accuracy can be increased without difficulty, only at the cost of some additional labor. It is applicable to an extensive group of related phenomena, such as vibrations in liquid columns, gas columns and electric surges.

APPENDIX I.

I. Symbols and Formulas.

For Rod in Longitudinal Stress.

L = length of rod,
F = cross-sectional area of rod,
ρ = density of rod material,
m = mass of rod = $LF\rho$,
E = modulus of longitudinal elasticity.

c = spring constant = force necessary to change the length by $1 = \frac{FE}{L}$,

a = velocity of propagation of a disturbance

$$= \sqrt{\frac{E}{\rho}} = \sqrt{\frac{c}{m}}\,L,$$

L/a = time-interval of single traversal of a disturbance

$$= \sqrt{\frac{m}{c}}\,.$$

For Helical Spring in Longitudinal Stress.

L = length of spring,
r = mean radius of coil,
d = wire diameter,
ρ = density of wire material,
n = number of coils,
$f = \frac{d^2\pi}{4}$ = cross-sectional area of wire,
J = polar moment of inertia of wire cross-section $= \frac{d^4\pi}{32}$,
m = mass of spring $= 2r\pi n \frac{d^2\pi}{4}\rho$ (neglecting slope of helix),
G = modulus of torsional elasticity,
c = spring constant = force necessary to alter the length by $1 = \frac{1}{2\pi} G \frac{J}{nr^3}$,

a = velocity of propagation of a disturbance along the spring axis $= \sqrt{\frac{c}{m}}\,L$,

L/a = interval of single traverse of a disturbance $= \sqrt{\frac{m}{c}}\,.$

t = time,
T = complete period of vibration
x = distance measured from one end of rod or spring,

M = concentrated mass,
$M/m = \sigma$ = ratio of concentrated mass to spring mass,
u = displacement,
$v = \frac{du}{dt}$ = velocity,
$s = \frac{du}{dt}$ = strain,
E = stress.

APPENDIX II.

Period of Vibration (Calculated).

According to Thamm's derivation (Ref. 4) the period of vibration T of two masses M_1 and M_2 attached to the ends of a spring having the mass m and spring constant c can be expressed by the equation:

$$T = \frac{2\pi}{\epsilon}\sqrt{\frac{m}{c}} = \frac{2\pi}{\epsilon}\frac{L}{a},$$

where the factor ϵ is a function of the ratio of spring mass to the concentrated masses and can be determined from the trigonometric equation:

$$\left(\epsilon - \frac{1}{\epsilon}\frac{m^2}{M_1 M_2}\right)\tan\epsilon = \frac{m}{M_1} + \frac{m}{M_1}$$

or

$$\left(\epsilon - \frac{1}{\epsilon}\frac{1}{\sigma_1\sigma_2}\right)\tan\epsilon = \frac{1}{\sigma_1} + \frac{1}{\sigma_2}.$$

The factor ϵ has an infinite number of values, of which the lowest value corresponding to the greatest value of T is the most important. These lowest values of ϵ and the greatest of T will be given for a number of particular cases:

(a) Spring fixed at one end, free at the other end. $M_1 = \infty$;

$$M_2 = 0;\ \epsilon = \frac{\pi}{2};\ T = 4\sqrt{\frac{m}{c}} = 4\frac{L}{a}.$$

(b) Spring fixed at both ends. $M_1 = \infty$; $M_2 = \infty$; $\epsilon = \pi$;

$$T = 2\sqrt{\frac{m}{c}} = 2\frac{L}{a}.$$

(c) Spring free at both ends. $M_1 = 0$; $M_2 = 0$; $\epsilon = \pi$;

$$T = 2\sqrt{\frac{m}{c}} = 2\frac{L}{a}.$$

(d) Spring mass negligible in comparison to the finite masses M_1 and M_2 attached to the two ends. $M_1 \gg m$;

$$M_2 \gg m;\ \epsilon = \sqrt{m\left(\frac{1}{M_1} + \frac{1}{M_2}\right)};$$

$$T = 2\pi\sqrt{\frac{M_1 M_2}{c(M_1 + M_2)}} = 2\pi\sqrt{\frac{M_1 M_2}{m(M_1 + M_2)}}\frac{L}{a}.$$

(e) Spring mass is negligible, one end is fixed. $M_1 \gg m$;

$$M_2 = \infty;\ \epsilon = \sqrt{\frac{m}{M_1}};\ T = 2\pi\sqrt{\frac{M_1}{c}} = 2\pi\sqrt{\frac{M}{m}}\frac{L}{a}.$$

(f) One end fixed. $M_1 = \infty;\ \frac{M}{m} = 4;\ \epsilon = 0.48;\ T = 13.1\frac{L}{a}.$

(g) One end fixed. $M_1 = \infty;\ \frac{M}{m} = 9;\ \epsilon = .325;\ T = 19.3\frac{L}{a}.$

(h) $\frac{M_1}{m} = 4;\ \frac{M_2}{m} = 9;\ \epsilon = .588;\ T = 10.7\frac{L}{a}.$

(i) Both ends fixed. $\frac{M}{m} = 12$ mass in the mid-point. This is equivalent to $\frac{M}{m} = 6;\ \epsilon = .398;\ T = 15.8\frac{L}{a}.$

APPENDIX III.

Change of Velocity of a Mass Assuming a Linear $\frac{ds}{dv}$ Relationship.

In the graphical construction of the change of velocity of the mass it has been assumed that the strain in the portion of the spring adjoining the mass is constant during an L/a time-interval or during a submultiple $(1/n)(L/a)$ thereof.

1. Assuming (Fig. 10) at the instant of releasing the mass $v_1 = 0$, $s = s_1$ (point 1 in the v–s diagram), and considering during the first L/a interval s_1 constant and during the second

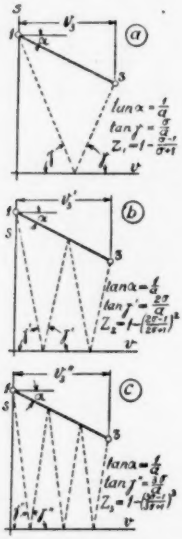
L/a interval s_3 constant, the velocity v_3 at the end of the first $2L/a$ interval will be:

$$v_3 = (s_1 + s_3)\frac{1}{\tan \gamma} = (s_1 + s_3)\frac{a}{\sigma},$$

considering that $s_1 - s_3 = v_3 \tan \alpha = v_3/a$, it can be written:

$$v_3 = s_1 a\left(1 - \frac{\sigma - 1}{\sigma + 1}\right) = s_1 \alpha Z_1.$$

FIG. 10.



Determination of velocity change with the value of strain assumed constant for (a) one L/a interval, (b) $(1/2)(L/a)$ interval, and (c) $(1/3)(L/a)$ interval.

2. A more close approximation will be attained by assuming a value of s being constant only for $(1/2)(L/a)$ interval, for which $\tan \gamma' = 2 \tan \gamma = 2\sigma/\alpha$ and

$$\Delta v = \frac{s}{\tan \gamma'} = \frac{sa}{2\sigma}.$$

Carrying out the summation for $4 \times (1/2)\ (L/a) = 2L/a$ intervals we obtain:

$$v_3' = s_1 a\left[1 - \left(\frac{2\sigma - 1}{2\sigma + 1}\right)^2\right] = s_1 a Z_2.$$

$$v = S_1 a Z.$$

$\sigma = \frac{M}{m}$	$\frac{1}{\sigma} = \frac{m}{M}$	$Z_1 = 1 - \frac{\sigma - 1}{\sigma + 1}$	$Z_2 = 1 - \left(\frac{2\sigma - 1}{2\sigma + 1}\right)^2$	$Z_3 = 1 - \left(\frac{3\sigma - 1}{3\sigma + 1}\right)^3$	$Z_n = 1 - e^{-2/\sigma}$
0.0	∞	2	0	2	1.000
0.1	10	1.818	0.556	1.156	0.9999
0.2	5	1.667	0.817	1.016	0.9999
0.3	3.333	1.538	0.937	1.000	0.999
0.333	3	1.500	0.960	1.000	0.998
0.4	2.5	1.428	0.988	0.999	0.996
0.5	2	1.333	1.000	0.992	0.982
0.6	1.667	1.250	0.992	0.977	0.965
0.667	1.5	1.200	0.980	0.963	0.950
0.7	1.428	1.176	0.972	0.955	0.942
0.8	1.25	1.111	0.945	0.931	0.918
0.9	1.111	1.053	0.918	0.903	0.891
1.0	1.000	1.000	0.890	0.875	0.865
2	0.5	0.667	0.640	0.635	0.632
3	0.333	0.500	0.490	0.488	0.487
4	0.25	0.400	0.395	0.394	0.393
5	0.2	0.333	0.330	0.330	0.330
6	0.167	0.286	0.284	0.285	0.283
7	0.143	0.250	0.249	0.249	0.247
8	0.125	0.222	0.222	0.220	0.220
9	0.111	0.200	0.200	0.200	0.200
10	0.1	0.182	0.180	0.180	0.180

3. A still more close approximation will be attained by assuming the value of s being constant only for $(1/3)\,(L/a)$ interval, for which $\tan \gamma'' = 3 \tan \gamma = 3\sigma/a$, and $\Delta v = sa/3\sigma$. Carrying out the summation for $6 \times (1/3)\,(L/a) = 2L/a$ intervals

$$v_3'' = s_1 a \left[1 - \left(\frac{3\sigma - 1}{3\sigma + 1} \right)^3 \right] = s_1 a Z_3.$$

4. The true value of v_3 will be obtained if the time duration $(1/n)(L/a)$ is made to approach zero, i.e., n is made to approach infinity, and the summation of the velocity increments so obtained is carried out for the $2n(1/n)\ (L/a) = 2L/a$ interval. Then it can be written:

$$v_3 = \lim_{n=\infty} s_1 a \left[1 - \left(\frac{n\sigma - 1}{n\sigma + 1} \right)^n \right] = s_1 a \left(1 - e^{-2/\sigma} \right) = s_1 a Z_n,$$

where $e = 2.71828$, the base of the natural logarithm. (Note: The value of v_3 can be also obtained directly from the law of momentum:

$$FEdt = Mdv \text{ whence } \frac{dv}{dt} = \frac{FE}{M},$$

considering that

$$s = s_1 - \frac{v}{a}; \qquad E = \frac{E}{\rho}. \qquad \frac{LF\rho}{LF} = a^2 \frac{m}{LF},$$

it can be written:

$$\frac{dv}{dt} = \frac{as}{\sigma\left(\frac{L}{a}\right)} = \frac{a}{\sigma\left(\frac{L}{a}\right)} \left(s_1 - \frac{v}{a} \right).$$

Integrating we obtain:

$$\int_0^{v_3} \frac{dv}{as_1 - v} = \int_0^{2L/a} \frac{dt}{\sigma\left(\frac{L}{a}\right)}$$

$$- \text{lognat} \left(\frac{as_1 - v_3}{as_1} \right) = \frac{2}{\sigma},$$

whence

$$v_3 = as_1(1 - e^{-2/\sigma}).)$$

The values of Z_1, Z_2, Z_3 and Z_n are tabulated in the adjoining table which shows that for values of $\sigma > 2$ the Z_1 and Z_2, and for values $\sigma > 0.2$ Z_3, differ less than 2 per cent from the true Z_n value.

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$$\text{Frequency} = d.12\,\frac{k}{W_L + C_1 W_S},$$

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SOME APPLICATIONS OF NORMAL COÖRDINATES TO ENGINEERING VIBRATION SYSTEMS.

BY

R. EKSERGIAN, M.S., M.E.E., Ph.D.

In this paper, consideration is given to the elementary theory and application of normal coördinates to some simple torsional systems. In small vibrations, the equations of motion are linear with constant coefficients, that is, the forces and couplings of the system are linear with respect to the coordinates. Further, the potential and kinetic energies are quadratic functions of the displacements and velocities, expressed in terms of the n generalized coördinates corresponding to the n degrees of freedom of the system. The choice of coordinates is immaterial in specifying the dynamic properties of the system. A particular class of coördinates, called normal coördinates, have the property of reducing to sums of squares, i.e., without cross product terms, in the expressions for kinetic and potential energies. This is a property that two quadratic functions can be simultaneously transformed to sums of squares of the parameters. The n normal coördinates have linear relations to the ordinary coördinates corresponding to the n degrees of freedom of the system.

Let the n ordinary coördinates, measured from their equilibrium position, be designated by $\theta_1, \theta_2, \cdots \theta_n$. In a normal vibration, corresponding to a natural frequency of the system, the displacements maintain fixed ratios with respect to each other, the amplitudes being either in or opposite in phase. The ratios of the displacements or amplitudes relative to the reference coördinate θ_1 when vibrating in any given mode at a natural frequency ω_s are designated by

$$\left(\frac{\theta_2}{\theta_1}\right)_{\omega_1} = k_{21}', \quad \left(\frac{\theta_3}{\theta_1}\right)_{\omega_1} = k_{31}', \quad \cdots \quad \left(\frac{\theta_n}{\theta_1}\right)_{\omega_1} = k_{n1}', \quad \textit{1st mode}$$

$$\left(\frac{\theta_2}{\theta_1}\right)_{\omega_2} = k_{21}'', \quad \left(\frac{\theta_3}{\theta_1}\right)_{\omega_2} = k_{31}'', \quad \cdots \quad \left(\frac{\theta_n}{\theta_1}\right)_{\omega_2} = k_{n1}'', \quad \textit{2d mode}$$

. . . . . .

$$\left(\frac{\theta_2}{\theta_1}\right)_{\omega_n} = k_{21}^{(n)}, \quad \left(\frac{\theta_3}{\theta_1}\right)_{\omega_n} = k_{31}^{(n)}, \quad \cdots \quad \left(\frac{\theta_n}{\theta_1}\right)_{\omega_n} = k_{n1}^{(n)}. \quad \textit{nth mode}$$

Normal coördinates characterize these various modes of vibration, corresponding to the natural frequencies $\omega_1, \omega_2, \cdots \omega_n$ of the system. Therefore, if ϕ corresponds to the reference coordinate, then any other coördinate of the system, when vibrating in this mode only, and corresponding to a natural frequency of the system, is $k_R\phi$. The subscript 1 is omitted, since θ_1 will be taken as the reference coördinate when vibrating in its particular mode.

Due to the linear properties of the system, the actual vibration can be considered as compounded from these modes of vibration. Since there are n modes corresponding to the n degrees of freedom and natural frequencies, the actual displacement of any coördinate θ_R in terms of normal coördinates $\phi_1, \phi_2 \cdots \phi_n$ is

$$\theta_R = k_R'\phi_1 + k_R''\phi_2 \cdots + k_R^{(n)}\phi_n.$$

When the system is expressed in terms of normal coordinates corresponding to the various modes of oscillation, since there can be no coupling between the various modes (with friction neglected), then in the expression of kinetic and potential (elastic) energy, cross product terms of the velocity and displacement coördinates disappear.

With normal coördinates, the expression for kinetic and potential energies take the simplified form,

$$T = 1/2[a_1\dot{\phi}_1^2 + a_2\dot{\phi}_2^2 \cdots + a_n\dot{\phi}_n^2],$$
$$V = 1/2[c_1\phi_1^2 + c_2\phi_2^2 \cdots + c_n\phi_n^2]$$

and the equations of free motion separate out to

$$a_1\ddot{\phi}_1 + c_1\phi_1 = 0, \qquad \text{where} \qquad \frac{c_1}{a_1} = \omega_1^2$$

$$a_2\ddot{\phi}_2 + c_2\phi_2 = 0, \qquad " \qquad \frac{c_2}{a_2} = \omega_2^2$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$a_n\ddot{\phi}_n + c_n\phi_n = 0 \qquad " \qquad \frac{c_n}{a_n} = \omega_n^2$$

and ω_1, ω_2 to ω_n correspond to the natural frequencies of the system.

In the case of forced vibrations, let $Q_1, Q_2, \cdots Q_n$ be the generalized forces applied to the normal coördinates, then, for the sth mode with natural frequency ω_s

$$a_s\ddot{\phi}_s + c_s\phi_s = Q_s e^{i(pt-\alpha_s)}$$

$$\therefore \phi_s = \frac{Q_s e^{i(pt-\alpha_s)}}{a_s(\omega_s^2 - p^2)}, \qquad \text{where} \qquad \omega_s^2 = \frac{c_s}{a_s}$$

and

$$a_s = I_1 + I_2 k_2^{(s)2} \cdots + I_n k_n^{(s)2},$$

where I_1 is the reference mass corresponding to θ_1 in the sth mode, and where the coördinates $\theta_2, \theta_3, \cdots \theta_n$ are taken as the coördinates of the masses $I_2, I_3, \cdots I_n$. The actual displacement for any coördinate θ_R is,

$$\theta_R = \sum_{s=1}^{s=n} \frac{k_R^{(s)} Q_s e^{i(pt-\alpha_s)}}{a_s(\omega_s^2 - p^2)}.$$

Certain systems have n degrees of freedom in which one of the n roots in the frequency equation $\Delta(\omega^2) = 0$, degenerates to zero, i.e., $\omega^2 = 0$. The mode corresponding to this degenerated frequency can be considered as a rigid motion of the system without torsion. Thus the motion of the system consists of $n - 1$ vibration modes and a rigid mode. In terms of normal coördinates, such systems are characterized by $n - 1$ vibration coördinates corresponding to the normal vibration modes and a rigid coördinate where the corresponding rigid mode has a degenerated natural frequency $\omega = 0$. Since the normal coördinates are orthogonal, cross product terms between the rigid and normal vibration coördinates in the expression of energy must also disappear.

The generalized momentum relative to the rigid coördinates is equal to the momentum for the rigid mode of the system. Since the amplitudes of the normal vibration coordinates, in free vibration of the system, are arbitrary, their coefficients in the expression of generalized momentum must be nil. This also shows the orthogonal relations between the rigid and normal coördinates in the disappearance of cross product terms in the expression of kinetic energy.

For the expression of internal stress and the elastic potential of the system, rigid coördinates do not appear. Thus,

while the kinetic energy includes terms with velocity squares in both normal (or vibrating) and rigid coördinates, the internal elastic potential only includes terms with squares of the normal (or vibration) coördinates. This is in accord with the degenerated frequency $\omega^2 = 0$ for the rigid mode.

In the calculation of non-resonance stresses normal coordinates offer simplification and a clearer insight as to the effect of the exciting forces. No vibration of a given mode can be excited when applied at one of its nodes, even though its frequency is near resonance with the given mode. Frequently, with a rapid convergence, the lower modes, with their corresponding natural frequencies and fixed ratios of amplitudes, are sufficient for a first approximation. Such modes are assumed calculated from a simplified analysis approximating the true modes. Obviously, since the actual displacements and stresses require a compounding of all the modes equal to the degrees of freedom of the system, the accuracy of the approximation depends upon the rapidity of the convergence of the higher modes under the excitation forces.

In the following analysis of torsional systems, consideration is given to their free and forced vibrations, and the calculation of stresses, with the application of normal coördinates and the effect of damping which introduces couplings between the modes.

1. TWO MASS TORSION SYSTEM WITH ELASTIC COUPLING.

Let us consider an elastic coupling between two masses I_1 and I_2 otherwise free. Let C = elastic constant of the coupling and θ_1 and θ_2 the coördinates of the respective masses. The equations of free vibration are

$$\left.\begin{aligned} I_1\ddot{\theta}_1 + c(\theta_1 - \theta_2) &= 0 \\ I_2\ddot{\theta}_2 + c(\theta_2 - \theta_1) &= 0 \end{aligned}\right\}. \tag{1}$$

With $\theta_1 = \theta_{10} \sin \omega t$, and $\theta_2 = \theta_{20} \sin \omega t$, then

$$\left.\begin{aligned} (c - \omega^2 I_1)\theta_{10} - c\theta_{20} &= 0 \\ -c\theta_{10} + (c - \omega^2 I_2)\theta_{20} &= 0 \end{aligned}\right\} \tag{2}$$

and the determinate, on eliminating the ratios of the amplitudes, is

$$\begin{vmatrix} c - \omega^2 I_1, & -c \\ -c, & c - \omega^2 I_2 \end{vmatrix} = 0. \tag{3}$$

This is the frequency equation and involves two roots for ω^2 of which one root degenerates to 0. This corresponds to two possible modes of oscillation, one of which corresponds to the system moving as if rigid. The frequency of the other mode is

$$\omega^2 = c\left(\frac{1}{I_1} + \frac{1}{I_2}\right). \tag{4}$$

Considering either equation of (2), we obtain for the ratio of amplitudes in the vibration mode

$$k = \frac{\theta_{20}}{\theta_{10}} = \frac{c - \omega^2 I_1}{c} = -\frac{I_1}{I_2}. \tag{5}$$

Let the normal and rigid coördinates be ϕ and ϕ_R which correspond to the normal modes discussed, so that

$$\theta_1 = \phi_R + \phi, \qquad \theta_2 = \phi_R + k\phi. \tag{6}$$

In substituting in equation (1) and adding,

$$\begin{aligned} (I_1 + I_2)\ddot{\phi}_R + (I_1 + kI_2)\ddot{\phi} &= 0 \\ \therefore (I_1 + I_2)\dot{\phi}_R &= c_R \end{aligned}, \tag{7}$$

which is the rigid momentum.

The kinetic energy of the system is

$$T = \tfrac{1}{2}(I_1 + I_2)\dot{\phi}_R^2 + \tfrac{1}{2}(I_1 + k^2 I_2)\dot{\phi}^2 + (I_1 + kI_2)\dot{\phi}_R\dot{\phi}. \tag{8}$$

The generalized momentum relative to the rigid coördinate is

$$\frac{\partial T}{\partial \dot{\phi}_R} = (I_1 + I_2)\dot{\phi}_R + (I_1 + kI_2)\dot{\phi} = c_R. \tag{9}$$

Since the amplitude of the vibration mode is arbitrary and independent of the rigid motion,

$$(I_1 + I_2)\dot{\phi}_R = c_R \qquad \text{and} \qquad I_1 + I_2 k = 0 \tag{10}$$

in agreement with (5) and (7). Thus the cross product terms in the expression of kinetic energy must disappear. Hence

$$T = \tfrac{1}{2}[(I_1 + I_2)\dot{\phi}_R^2 + (I_1 + k^2 I_2)\dot{\phi}^2]. \tag{11}$$

The elastic energy is $V = \frac{1}{2}c(\theta_2 - \theta_1)^2$, so that

$$V = \tfrac{1}{2}c(k-1)^2\phi^2 \qquad \text{and} \qquad \frac{\partial V}{\partial \phi} = c(k-1)^2\phi. \tag{12}$$

Forced Oscillation: If a harmonic impressed force $P \sin pt$ is applied to I_2 then its virtual work is $P \sin pt(\delta\phi_R + k\delta\phi)$ and the Lagrangian equations of motion are

$$\begin{aligned} (I_1 + I_2)\ddot{\phi}_R &= P \sin pt \\ (I_1 + I_2k^2)\ddot{\phi} + c(k-1)^2\phi &= Pk \sin pt. \end{aligned} \tag{13}$$

The natural frequency of the vibration mode is

$$\omega^2 = \frac{c(k-1)^2}{I_1 + k^2 I_2} \qquad \text{and since} \qquad k = -\frac{I_1}{I_2}$$

the above reduces to expression (4). Hence,

$$\left.\begin{aligned} \ddot{\phi}_R &= \frac{P \sin pt}{I_1 + I_2} \\ \ddot{\phi} + \omega^2\phi &= \frac{Pk \sin pt}{I_1 + k^2 I_2} \end{aligned}\right\}, \tag{14}$$

therefore

$$\phi_R = -\frac{P \sin pt}{p^2(I_1 + I_2)}, \qquad \phi = \frac{Pk \sin pt}{(\omega^2 - p^2)(I_1 + k^2 I_2)}, \tag{15}$$

the torque in the shaft Φ_s is $c(\theta_2 - \theta_1) = c(k-1)\phi$,

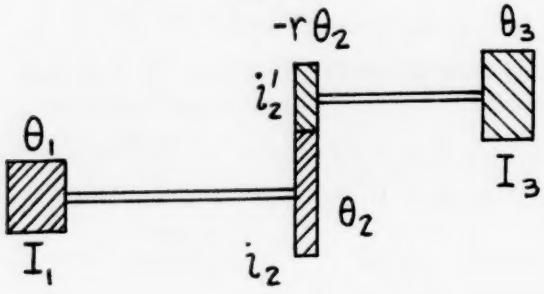
$$\therefore \Phi_s = \left[\frac{c(k-1)}{I_1 + k^2 I_2}\right]\frac{Pk \sin pt}{\omega^2 - p^2}. \tag{16}$$

On substituting for $k = -I_1/I_2$,

$$\Phi_s = \left[\frac{I_1}{I_1 + I_2}\right]\frac{P \sin pt}{(1 - (p/\omega)^2)}. \tag{17}$$

2. SINGLE REDUCTION GEARED SYSTEM: FIG. 1.

FIG. 1.



Let r gear ratio, so that $I_2 = i_2 + r^2 i_2'$. The kinetic and potential energies are

$$T = \tfrac{1}{2}[I_1\dot{\theta}_1^2 + I_2\dot{\theta}_2^2 + I_3\dot{\theta}_3^2] \tag{18}$$

and

$$V = \tfrac{1}{2}c_{12}(\theta_1 - \theta_2)^2 + \tfrac{1}{2}c_{23}(\theta_3 + r\theta_2)^2. \tag{19}$$

The equations of motion are, therefore,

$$\left.\begin{aligned} I_1\ddot{\theta}_1 + c_{12}(\theta_1 - \theta_2) &= 0 \\ I_2\ddot{\theta}_2 + c_{12}(\theta_2 - \theta_1) + rc_{23}(\theta_3 + r\theta_2) &= 0 \\ I_3\ddot{\theta}_3 + c_{23}(\theta_3 + r\theta_2) &= 0 \end{aligned}\right\}. \tag{20}$$

With $\theta_1 = \theta_{10}e^{i\omega t}$, $\theta_2 = \theta_{20}e^{i\omega t}$ and $\theta_3 = \theta_{30}e^{i\omega t}$, on eliminating the ratio of amplitudes, we obtain the frequency equation. The frequency of the two vibration modes is determined from the determinate,

$$\begin{vmatrix} c_{12} - \omega^2 I_1 & -c_{12} & 0 \\ -c_{12} & (r^2 c_{23} + c_{12} - \omega^2 I_2) & rc_{23} \\ 0 & rc_{23} & c_{23} - \omega^2 I_3 \end{vmatrix} = 0. \tag{21}$$

The coördinates θ_1, θ_2 and θ_3, when expressed in terms of rigid and normal coördinates, are

$$\left.\begin{aligned} \theta_1 &= \phi_R + \phi_1 + \phi_2 \\ \theta_2 &= \phi_R + k_2'\phi_1 + k_2''\phi_2 \\ \theta_3 &= -r\phi_R + k_3'\phi_1 + k_3''\phi_2 \end{aligned}\right\}, \tag{22}$$

where ϕ_R corresponds to the rigid motion and ϕ_1 and ϕ_2 are the coördinates corresponding to the first and second modes, with frequencies ω_1 and ω_2.

On substituting in (18) and (19) the kinetic and elastic energy is expressed in terms of the normal coördinates and the ratios of amplitudes k for the two vibration modes.

To show the elimination of the cross product terms, between the rigid and vibration coördinates, we have for the momentum of the rigid motion

$$\frac{\partial T}{\partial \dot{\phi}_R} = (I_1 + I_2 + r^2 I_3)\dot{\phi}_R + (I_1 + I_2 k_2' - I_3 r k_3')\dot{\phi}_1 + (I_1 + I_2 k_2'' - I_3 r k_3'')\dot{\phi}_2 = c_R, \tag{23}$$

where $c_R = (I_1 + I_2 + r^2 I_3)\dot{\phi}_R$ = the rigid momentum. Hence, since ϕ_2 and ϕ_3 are arbitrary in their amplitudes,

$$I_1 + I_2 k_2' - I_3 r k_3' = 0, \qquad I_1 + I_2 k_2'' - I_3 r k_3'' = 0. \quad (24)$$

For the elimination of the cross product terms in the vibration coördinates, if ω is the frequency of either mode, where the amplitude ratios for either mode are given by k, then from (20) the vibration of either mode is, i.e., with $\omega = \omega_1$ and $\omega = \omega_2$, respectively,

$$\left.\begin{aligned} -\omega^2 I_1 + c_{12}(1 - k_2) &= 0 \\ -\omega^2 I_2 k_2 + c_{12}(k_2 - 1) + r c_{23}(k_3 + r k_2) &= 0 \\ -\omega^2 I_3 k_3 + c_{23}(k_3 + r k_2) &= 0 \end{aligned}\right\}. \quad (25)$$

On adding the first two and subtracting the third after multiplying by r we also obtain the relations (8).

On multiplying the second and third expressions by the amplitude ratios k for the *other* mode, then,

$$\begin{aligned} -\omega_1^2[I_1 + I_2 k_2' k_2'' + I_3 k_3' k_3''] \\ + c_{12}(1 - k_2' - k_2'' + k_2' k_2'') \\ + c_{23}[r(k_3' k_2'' + k_2' k_3'') + r^2 k_2' k_2'' + k_3' k_3''] = 0 \end{aligned}$$

and

$$\begin{aligned} -\omega_2^2[I_1 + I_2 k_2' k_2'' + I_3 k_3' k_3''] \\ + c_{12}(1 - k_2' - k_2'' - k_2' k_2'') \\ + c_{23}[r(k_3' k_2'' + k_2' k_3'') + r^2 k_2' k_2'' + k_3' k_3''] = 0. \end{aligned}$$

Since $\omega_1 \neq \omega_2$

$$I_1 + I_2 k_2' k_2'' + I_3 k_3' k_3'' = 0$$

and

$$\begin{aligned} c_{12}(1 - k_2' - k_2'' + k_2' k_2'') \\ + c_{23}[r(k_3' k_2'' + k_2' k_3'') + r^2 k_2' k_2'' + k_3' k_3''] = 0, \end{aligned}$$

thus eliminating the cross product terms of the normal coordinates.

The kinetic and potential energies are thus expressed only in terms of the squares of the velocity and displacement coördinates.

$$\begin{aligned} T = \tfrac{1}{2}[I_1 + I_2 + I_3 r^2]\dot{\phi}_R^2 + \tfrac{1}{2}[I_1 + I_2 k_2'^2 + I_3 k_3'^2]\dot{\phi}_1^2 \\ + \tfrac{1}{2}[I_1 + I_2 k_2''^2 + I_3 k_3''^2]\dot{\phi}_2^2 \quad (26) \end{aligned}$$

and

$$V = \tfrac{1}{2}c_{12}(1 - k_2')^2\phi_1^2 + \tfrac{1}{2}c_{12}(1 - k_2'')^2\phi_2^2 + \tfrac{1}{2}c_{23}(k_3' + rk_2')^2\phi_1^2 + \tfrac{1}{2}c_{23}(k_3'' + rk_2'')^2\phi_2^2. \quad (27)$$

Forced Oscillation: Assume $\Phi \sin pt$ applied to mass I_3; then since

$$\Phi\delta\theta_3 = \Phi[-r\delta\phi_R + k_3'\delta\phi_1 + k_3''\delta\phi_2],$$

the equations of motion are

$$\left.\begin{aligned} \ddot{\phi}_R &= \frac{-\Phi r \sin pt}{I_1 + I_2 + I_3r^2} \\ \ddot{\phi}_1 + \omega_1^2\phi_1 &= \frac{\Phi k_3' \sin pt}{I_1 + I_2k_2'^2 + I_3k_3'^2} \\ \ddot{\phi}_2 + \omega_2^2\phi_2 &= \frac{\Phi k_3'' \sin pt}{I_1 + I_2k_2''^2 + I_3k_3''^2} \end{aligned}\right\}, \quad (28)$$

where it is to be noted from equation (25) for either mode, that

$$\left.\begin{aligned} \frac{c_{12}(1 - k_2')^2 + c_{23}(k_3' + rk_2')^2}{I_1 + I_2k_2'^2 + I_3k_3'^2} &= \omega_1^2 \\ \frac{c_{12}(1 - k_2'')^2 + c_{23}(k_3'' + rk_2'')^2}{I_1 + I_2k_2''^2 + I_3k_3''^2} &= \omega_2^2 \end{aligned}\right\} \quad (29)$$

and ω_1^2 and ω_2^2 are the roots of the determinate (21). The normal forced vibrations are, therefore,

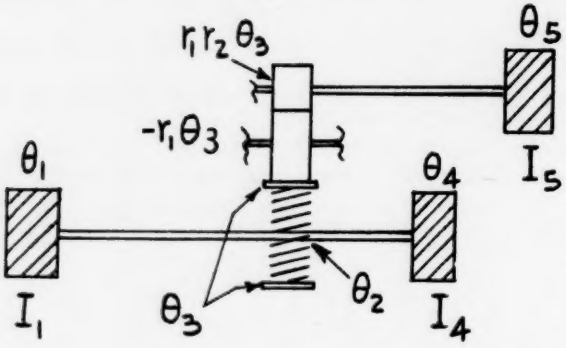
$$\left.\begin{aligned} \phi_R &= \frac{r\Phi \sin pt}{p^2[I_1 + I_2 + I_3r^2]} \\ \phi_1 &= \frac{\Phi k_3' \sin pt}{(\omega_1^2 - p^2)[I_1 + I_2k_2'^2 + I_3k_3'^2]} \\ \phi_2 &= \frac{\Phi k_3'' \sin pt}{(\omega_2^2 - p^2)[I_1 + I_2k_2''^2 + I_3k_3''^2]} \end{aligned}\right\}. \quad (30)$$

The stress in the pinion shaft is

$$c_{23}(\theta_3 + r\theta_2) = c_{23}[(k_3' + rk_2')\phi_1 + (k_3'' + rk_2'')\phi_2]. \quad (31)$$

3. FLEXIBLE GEARED SYSTEM: FIG. 2.

FIG. 2.



Consider a double reduction gear system, with a torsional spring in the main gear wheel, with the gear ratio r_1 for the first gear pair and r_2 for the second. The coördinate for the shaft at the main gear is θ_2 and for the gear rim θ_3. The inertias of the gears are neglected in comparison with the rotor inertias $I_1 I_4$ and I_5 with coördinates θ_1, θ_4 and θ_5.

The kinetic and potential energies are

$$T = \tfrac{1}{2}(I_1\dot{\theta}_1^2 + I_4\dot{\theta}_4^2 + I_5\dot{\theta}_5^2), \tag{32}$$

$$V = \tfrac{1}{2}[c_{12}(\theta_1 - \theta_2)^2 + c_{24}(\theta_4 - \theta_2)^2 + c_{35}(\theta_5 - r_1 r_2\theta_3)^2 + c_{23}(\theta_3 - \theta_2)^2], \tag{33}$$

so that the equations of motion are

$$\left.\begin{aligned} I_1\ddot{\theta}_1 + c_{12}(\theta_1 - \theta_2) &= 0 \\ I_4\ddot{\theta}_4 + c_{24}(\theta_4 - \theta_2) &= 0 \\ I_5\ddot{\theta}_5 + c_{35}(\theta_5 - r_1 r_2\theta_3) &= 0 \\ -\frac{\partial V}{\partial \theta_2} = c_{12}(\theta_1 - \theta_2) + c_{24}(\theta_4 - \theta_2) & \\ + c_{23}(\theta_3 - \theta_2) &= 0 \\ -\frac{\partial V}{\partial \theta_3} = -r_1 r_2 c_{35}(\theta_5 - r_1 r_2\theta_3) + c_{23}(\theta_3 - \theta_2) &= 0 \end{aligned}\right\}. \tag{34}$$

On elimination of θ_2 and θ_3 and then eliminating the ratio of amplitudes in the remaining coördinates $\theta_1 = \theta_{10}e^{i\omega t}$, $\theta_4 = \theta_{40}e^{i\omega t}$ and $\theta_5 = \theta_{50}e^{i\omega t}$, the frequency equation is readily

obtained. The system has two vibration frequencies ω_1 and ω_2 and ϕ_1 and ϕ_2 are the corresponding normal modes. Hence in terms of rigid and normal coördinates,

$$\left.\begin{aligned} \theta_1 &= \phi_R + \phi_1 + \phi_2 \\ \theta_4 &= \phi_R + k_4'\phi_1 + k_4''\phi_2 \\ \theta_5 &= r_1r_2\phi_R + k_5'\phi_1 + k_5''\phi_2 \\ \theta_2 &= \phi_R + k_2'\phi_1 + k_2''\phi_2 \\ \theta_3 &= \phi_R + k_3'\phi_1 + k_3''\phi_2 \end{aligned}\right\}, \tag{35}$$

where the k''s are the ratios of the amplitudes relative to I_1 for mode 1, and k'' correspond to mode 2.

The cross product terms, after substituting (35) in the expressions for kinetic and potential energies, equations (32) and (33), are eliminated as follows. On equating the generalized momentum relative to the rigid coördinate, to the momentum for the rigid mode and noting ϕ_1 and ϕ_2 have arbitrary amplitudes, the coefficients of the cross product terms of rigid and normal coördinates are eliminated. In like manner, considering the vibration of either normal mode in equation (34) and multiplying the second, third, fourth and fifth equations throughout by the amplitude ratios k_4, k_5, k_2 and k_3 of the other mode and adding all the equations, then since $\omega_1 \neq \omega_2$, the coefficients of the cross product terms of the normal coördinates in the expressions of kinetic and potential energies are seen to disappear.

Again, considering equation (34) for the vibration of either mode it is easy to see that

$$\begin{aligned} \omega_1^2[I_1 + I_4k_4'^2 + I_5k_5'^2] &= c_{12}(1 - k_2')^2 + c_{24}(k_4' - k_2')^2 \\ &\quad + c_{35}(k_5' - r_1r_2k_3')^2 + c_{23}(k_3' - k_2')^2 \end{aligned} \tag{36}$$

and

$$\begin{aligned} \omega_2^2[I_1 + I_4k_4''^2 + I_5k_5''^2] &= c_{12}(1 - k_2'')^2 + c_{24}(k_4'' - k_2'')^2 \\ &\quad + c_{35}(k_5'' - r_1r_2k_3'')^2 + c_{23}(k_3'' - k_2'')^2. \end{aligned} \tag{37}$$

The expression for kinetic and elastic energies in terms of the rigid and normal coördinates on substituting (35) in (32) and (33) and noting the elimination of the cross product

terms, is

$$T = \tfrac{1}{2}[I_1 + I_4 + r_1^2 r_2^2 I_5]\dot{\phi}_R^2 + \tfrac{1}{2}[I_1 + I_4 k_4'^2 + I_5 k_5'^2]\dot{\phi}_1^2 + \tfrac{1}{2}[I_1 + I_4 k_4''^2 + I_5 k_5''^2]\dot{\phi}_2^2, \quad (38)$$

$$V = \tfrac{1}{2}[c_{12}(1 - k_2')^2 + c_{24}(k_4' - k_2')^2 + c_{35}(k_5' - r_1 r_2 k_3')^2 + c_{23}(k_3' - k_2')^2]\phi_1^2 + \tfrac{1}{2}[c_{12}(1 - k_2'')^2 + c_{24}(k_4'' - k_2'')^2 + c_{35}(k_5'' - r_1 r_2 k_3'')^2 + c_{23}(k_3'' - k_2'')^2]\phi_2^2. \quad (39)$$

Forced Oscillation: Assume $\Phi_4 \sin pt$ acting on mass I_4 and $\Phi_5 \sin (pt - \alpha)$ acting on mass I_5. Then since

$$\Phi_4 \sin pt \cdot \delta\theta_4 = \Phi_4[\delta\phi_R + k_4'\delta\phi_1 + k_4''\delta\phi_2] \sin pt$$

and

$$\Phi_5 \sin (pt - \alpha)\delta\theta_5 = \Phi_5[r_1 r_2 \delta\phi_R + k_5'\delta\phi_1 + k_5''\delta\phi_2] \sin (pt - \alpha).$$

The equations of motion, noting the relations (36) and (37), are

$$\left.\begin{aligned} \ddot{\phi}_R &= \frac{\Phi_4 \sin pt + r_1 r_2 \Phi_5 \sin (pt - \alpha)}{I_1 + I_4 + r_1^2 r_2^2 I_5} \\ \ddot{\phi}_1 + \omega_1^2 \phi_1 &= \frac{k_4'\Phi_4 \sin pt + k_5'\Phi_5 \sin (pt - \alpha)}{I_1 + I_4 k_4'^2 + I_5 k_5'^2} \\ \ddot{\phi}_2 + \omega_2^2 \phi_2 &= \frac{k_4''\Phi_4 \sin pt + k_5''\Phi_5 \sin (pt - \alpha)}{I_1 + I_4 k_4''^2 + I_5 k_5''^2} \end{aligned}\right\}. \quad (40)$$

Neglecting the free vibrations which are assumed damped out after the initial conditions, the steady forced vibrations are

$$\left.\begin{aligned} \phi_R &= -\frac{\Phi_R \sin (pt - \alpha_R)}{p^2[I_1 + I_4 + r_1^2 r_2^2 I_5]} \\ \phi_1 &= \frac{\Phi_{11} \sin (pt - \alpha_1)}{\omega_1^2(1 - p^2/\omega_1^2)[I_1 + I_4 k_4'^2 + I_5 k_5''^2]} \\ \phi_2 &= \frac{\Phi_{22} \sin (pt - \alpha_2)}{\omega_2^2(1 - p^2/\omega_2^2)[I_1 + I_4 k_4''^2 + I_5 k_5''^2]} \end{aligned}\right\}, \quad (41)$$

where

$$\phi_R = [(\Phi_4 + r_1 r_2 \Phi_5 \cos\alpha)^2 + (r_1 r_2 \Phi_5 \sin\alpha)^2]^{1/2},$$

$$\alpha_R = \tan^{-1} \frac{r_1 r_2 \Phi_5 \sin\alpha}{\Phi_4 + r_1 r_2 \Phi_5 \cos\alpha},$$

$$\Phi_{11} = [(k'\Phi_4 + k_5'\Phi_5 \cos\alpha)^2 + (k_5'\Phi_5 \sin\alpha)^2]^{1/2},$$

$$\alpha_1 = \tan^{-1} \frac{k_5'\Phi_5 \sin\alpha}{k'\Phi_4 + k_5'\Phi_5 \cos\alpha}, \qquad \text{etc.}$$

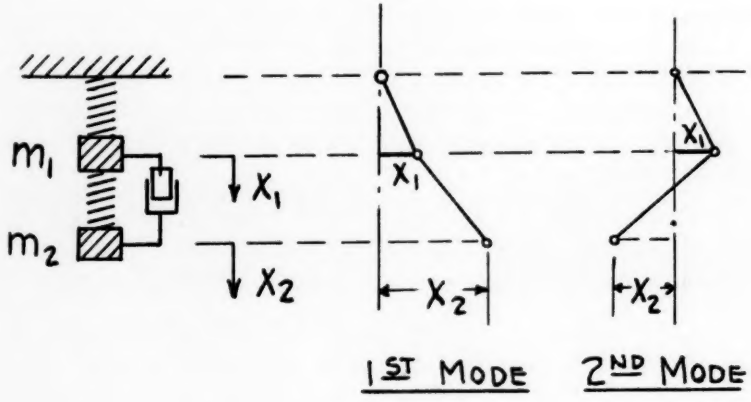
Stress in the Elastic Gear Wheel: To illustrate the computation of stresses in shafting or couplings, consider the stress in the elastic gear,

$$\begin{aligned} c_{23}(\theta_2 - \theta_3) &= c_{23}(k_2' - k_3')\phi_1 + c_{23}(k_2'' - k_3'')\phi_2 \\ &= \frac{c_{23}(k_2' - k_3')\Phi_{11} \sin(pt - \alpha_1)}{\omega_1^2(1 - p^2/\omega_1^2)(I_1 + I_4 k_4'^2 + I_5 k_5'^2)} \\ &\quad + \frac{c_{23}(k_2'' - k_3'')\Phi_{22} \sin(pt - \alpha_2)}{\omega_2^2(1 - p^2/\omega_2^2)(I_1 + I_4 k_4''^2 + I_5 k_5''^2)}, \end{aligned} \tag{42}$$

which shows the stresses in the coupling for the different modes must be vectorially added.

4. SYSTEMS WITH DAMPING: FIG. 3.

FIG. 3.



In the case of dissipative systems, normal coördinates between the potential and kinetic energies are selected, i.e., which would appear as the sum of squares of the displacement and velocity coördinates as if no dissipation function existed. With damping, in general, we have friction couplings between these normal coördinates. The damping forces, if small, have little effect on the frequencies of the free normal modes and the amplitudes of the other modes relative to any given mode are small, while the phases of these other modes are at quadrature with any given mode.

Referring to Fig. 3, if x_1 and x_2 are the displacements measured from their equilibrium position and c and c_{12} the spring constants, then the kinetic and potential energies are

$$T = \tfrac{1}{2}m_1\dot{x}_1^2 + \tfrac{1}{2}m_2\dot{x}_2^2,$$
$$V = \tfrac{1}{2}cx_1^2 + \tfrac{1}{2}c_{12}(x_2 - x_1)^2.$$

Without damping, the free motion is

$$\left.\begin{aligned} m_1\ddot{x}_1 + c_{12}(x_1 - x_2) + cx_1 &= 0 \\ m_2\ddot{x}_2 + c_{12}(x_2 - x_1) &= 0 \end{aligned}\right\}. \tag{43}$$

If $x_1 = x_{10}e^{i\omega t}$, $x_2 = x_{20}e^{i\omega t}$, then on substituting in (43) and eliminating the ratios of the amplitudes, we obtain the frequency equation

$$\begin{vmatrix} c_{12} + c - \omega^2 m_1 & -c_{12} \\ -c_{12} & c_{12} - \omega^2 m_2 \end{vmatrix} = 0. \tag{44}$$

From which the roots ω_1^2 and ω_2^2 are obtained and which correspond to the frequencies of the respective modes.

Let $k' = x_2/x_1$ when vibrating in the first mode at frequency ω_1, and $k'' = x_2/x_1$, when vibrating in the second mode at frequency ω_2, let the coördinate of m_1 be the reference coordinate. If ξ and η are the normal coördinates for these modes, then

$$x_1 = \xi + \eta, \qquad x_2 = k'\xi + k''\eta. \tag{45}$$

From equation (43) considering the vibration of the first mode only and multiplying the second equation by k''

$$[-\omega_1^2 m_1 + (c_{12} + c)] - c_{12}k' = 0,$$
$$[-\omega_1^2 m_2 + c_{12}]k'k'' - c_{12}k'' = 0,$$
$$\therefore -\omega_1^2(m_1 + m_2k'k'') + [c_{12} + c - c_{12}(k' + k'') + c_{12}k'k''] = 0.$$

Likewise, from consideration of the second mode, after multiplying the second equation by k',

$$-\omega_2^2(m_1 + m_2k''k') + [c_{12} + c - c_{12}(k' + k'') + c_{12}k'k''] = 0.$$

Since

$$\omega_1^2 \neq \omega_2^2,$$
$$m_1 + m_2k''k' = 0,$$
$$c_{12} + c - c_{12}(k' + k'') + c_{12}k'k'' = 0,$$

which shows the elimination of the cross product terms when substituting (45) in T and V.

Hence

$$T = \tfrac{1}{2}[m_1 + m_2k'^2]\dot{\xi}^2 + \tfrac{1}{2}[m_1 + m_2k''^2]\dot{\eta}^2, \tag{46}$$

$$V = \tfrac{1}{2}[c_{12}(k' - 1)^2 + c]\xi^2 + \tfrac{1}{2}[c_{12}(k'' - 1)^2 + c]\eta^2. \tag{47}$$

Damping Coupling: The friction coupling, in a first approximation, can be derived from a dissipation function F as given by Rayleigh,

$$-\frac{\partial F}{\partial \dot{x}_1} = K_{12}(\dot{x}_2 - \dot{x}_1) \quad \text{acting on } m_1$$

$$-\frac{\partial F}{\partial \dot{x}_2} = K_{12}(\dot{x}_1 - \dot{x}_2) \quad \text{acting on } m_2$$

$$\therefore F = \tfrac{1}{2}K_{12}(\dot{x}_2 - \dot{x}_1)^2. \tag{48}$$

When expressed in normal coördinates, then

$$T = \tfrac{1}{2}[a_1\dot{\xi}^2 + a_2\dot{\eta}^2], \qquad V = \tfrac{1}{2}[c_1\xi^2 + c_2\eta^2]$$

and

$$F = \tfrac{1}{2}[b_1\dot{\xi}^2 + b_2\dot{\eta}^2] + b_{12}\dot{\xi}\dot{\eta},$$

where

$$a_1 = m_1 + m_2k'^2, \qquad a_2 = m_1 + m_2k''^2$$
$$c_1 = c_{12}(k' - 1)^2 + c, \qquad c_2 = c_{12}(k'' - 1)^2 + c$$
$$b_1 = K_{12}(k' - 1)^2, \qquad b_2 = K_{12}(k'' - 1)^2,$$
$$b_{12} = K_{12}(k' - 1)(k'' - 1).$$

On substituting for ω, ω_1 and ω_2 and for k, k' and k'', corresponding to either mode in equation 43, and adding, after

multiplying the second equation by k, we obtain

$$\frac{c_1}{a_1} = \omega_1^2 \qquad \text{and} \qquad \frac{c_2}{a_2} = \omega_2^2$$

and these are the roots of the determinate (44). Further, ω_1 and ω_2 are the frequencies of the respective free modes of vibration without friction.

The equations of free vibration with damping are

$$\left.\begin{aligned} a_1\ddot{\xi} + b_1\dot{\xi} + c_1\xi + b_{12}\dot{\eta} &= 0 \\ a_2\ddot{\eta} + b_2\dot{\eta} + c_2\eta + b_{12}\dot{\xi} &= 0 \end{aligned}\right\} \tag{49}$$

and with

$$\xi = Ae^{\lambda t}, \qquad \eta = Be^{\lambda t},$$

we obtain the frequency equation,

$$\lambda^4 + 2\lambda^3(r_1 + r_2) + \lambda^2(\omega_1^2 + \omega_2^2) + 2\lambda(r_2\omega_1^2 + r_1\omega_2^2) + \omega_1^2\omega_2^2 = 0,$$

where $r_1 = b_1/2a_1$, and $r_2 = b_2/2a_2$, and second powers of b_1, b_2 and b_{12} are neglected. The complex roots are

$$|\lambda_1, \lambda_2| = -r_1 \pm i\omega_1, \qquad \text{and} \qquad |\lambda_3, \lambda_4| = -r_2 + i\omega_2.$$

Considering the damping small, the ratios of amplitudes are

$$B/A = \pm\frac{i\omega_1}{\omega_1^2 - \omega_2^2}\cdot\frac{b_{12}}{a_2} \quad \text{relative to 1st mode,}$$

$$A/B = \pm\frac{i\omega_2}{\omega_2^2 - \omega_1^2}\cdot\frac{b_{12}}{a_1} \quad \text{relative to 2d mode,}$$

so that the general solution is

$$\left.\begin{aligned} \xi &= (\omega_1^2 - \omega_2^2)c_1e^{-r_1t}\sin(\omega_1t + \alpha) \\ &\qquad + \frac{b_{12}}{a_1}\omega_2c_2e^{-r_2t}\sin(\omega_2t + \pi/2 + \beta) \\ \eta &= \frac{b_{12}}{a_2}\omega_1c_1e^{-r_1t}\sin(\omega_1t + \pi/2 + \alpha) \\ &\qquad + (\omega_2^2 - \omega_1^2)c_2e^{-r_2t}\sin(\omega_2t + \beta) \end{aligned}\right\}. \tag{50}$$

Forced Oscillation: Assuming a torque $P_1\cos pt$ acting on mass m_1, the system approximates an engine of rotational

inertia m_1 with its elastic shaft and spring constant c and an attacked vibration absorber of mass m_2, spring constant c_{12} and friction coupling K_{12}. Since

$$P_1 \cos pt(\delta\xi + \delta\eta) = \delta W,$$

the generalized forces applied to the normal coördinates, ξ and η, are

$$\Phi_\xi = P_1 \cos pt \qquad \text{and} \qquad \Phi_\eta = P_1 \cos pt.$$

Therefore, the equations of forced vibartion are

$$\left.\begin{aligned} a_1\ddot{\xi} + b_1\dot{\xi} + c_1\xi + b_{12}\dot{\eta} &= P_1 e^{ipt} \\ a_2\ddot{\eta} + b_2\dot{\eta} + c_2\eta + b_{12}\dot{\xi} &= P_1 e^{ipt} \end{aligned}\right\}. \tag{51}$$

If we let $\xi = \xi_0 e^{ipt}$ and $\eta = \eta_0 e^{ipt}$, then

$$\begin{aligned} (-p^2a_1 + ipb_1 + c_1)\xi_0 + ipb_{12}\eta_0 &= P_1 \\ ipb_{12}\xi_0 + (-p^2a_2 + ipb_2 + c_2)\eta_0 &= P_1. \end{aligned} \tag{52}$$

Noting the natural frequencies, corresponding to the normal modes, are $\omega_1^2 = \frac{c_1}{a_1}$ and $\omega_2^2 = \frac{c_2}{a_2}$, the above reduces to

$$\begin{aligned} \left[(\omega_1^2 - p^2) + ip\frac{b_1}{a_1}\right]\xi_0 + ip\frac{b_{12}}{a_1}\eta_0 &= P_1/a_1 \\ ip\frac{b_{12}}{a_2}\xi_0 + \left[(\omega_2^2 - p^2) + ip\frac{b_2}{a_2}\right]\eta_0 &= P_1/a_2. \end{aligned} \tag{53}$$

With small damping, the squares of the damping coefficients may be neglected, so that the determinate

$$\begin{vmatrix} (\omega_1^2 - p^2) + ip\dfrac{b_1}{a_1}, & \dfrac{ipb_{12}}{a_1} \\ ip\dfrac{b_{12}}{a_2}, & (\omega_2^2 - p^2) + ip\dfrac{b_2}{a_2} \end{vmatrix} \tag{54}$$

reduces to

$$i\omega_1(\omega_2^2 - \omega_1^2)\frac{b_1}{a_1} \qquad \text{when} \qquad p = \omega_1,$$

$$i\omega_2(\omega_1^2 - \omega_2^2)\frac{b_2}{a_2} \qquad \text{when} \qquad p = \omega_2,$$

therefore,

$$\left.\begin{aligned} |\xi_0| &= P_1\left[\frac{1}{i\omega_1 b_1} + \frac{(b_2 - b_{12})}{a_2 b_1(\omega_2^2 - \omega_1^2)}\right] \\ |\eta_0| &= P_1\frac{(b_1 - b_{12})}{a_2 b_1(\omega_2^2 - \omega_1^2)} \end{aligned}\right\} \quad \text{when} \quad p = \omega_1 \qquad (55)$$

and

$$\left.\begin{aligned} |\xi_0| &= \frac{P_1(b_2 - b_{12})}{a_1 b_2(\omega_1^2 - \omega_2^2)} \\ |\eta_0| &= P_1\left[\frac{1}{i\omega_2 b_2} + \frac{(b_1 - b_{12})}{a_1 b_2(\omega_1^2 - \omega_2^2)}\right] \end{aligned}\right\} \quad \text{when} \quad p = \omega_2. \qquad (56)$$

These are the conditions of resonance with either mode of oscillation. The damping couplings between the excited and non-excited modes are small being reduced below their static deflections. Therefore, when the applied forces are at resonance with either mode, a sufficient approximation is

$$|\xi_0| = \frac{P_1}{\omega_1 b_1} e^{-i\pi/2} \qquad \text{and} \qquad |\eta_0| = \frac{P_1}{\omega_2 b_2} e^{-i\pi/2}$$

$$\therefore \xi = \frac{P_1 e^{i(p_1 t - \pi/2)}}{\omega_1 b_1}, \qquad \eta = \frac{P_1 e^{i(p_2 t - \pi/2)}}{\omega_2 b_2} \qquad (57)$$

when $p_1 = \omega_1$, and $p_2 = \omega_2$, respectively.

Thus the displacements are at quadrature with the exciting force and the damping forces absorb the exciting forces. Moreover, the configuration of the system at resonance with a given mode is very little affected from that of its free vibration. Thus at resonance it is only necessary to equate the work of the exciting forces to that of the damping forces at quadrature, acting on the displacements whose ratios conform with the configuration of the excited mode.

Acknowledgment is due to Dr. T. D. Cope and Dr. E. E. Witmer for criticisms and the reviewing of the paper.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

CELLULOSE ACETATE SHEETING FOR PROTECTION OF DOCUMENTS.

The process of laminating paper with transparent cellulose acetate sheeting, developed several years ago at the Bureau as a preservative measure, has come into considerable use.

The first application of the process in the documentary field was by the National Archives of the United States. It was chosen as the best, and in fact, the only feasible means of laminating the vast number of documents which require such treatment received by the National Archives. The process is speedy and has many advantages. The sheeting is applied by means of heat and pressure, no adhesive being used, and a homogeneous unit is obtained, as the cellulose sheeting is actually forced into the pores of the paper. The acetate sheeting used is very stable, has high transparency and strength, is very thin, is easily cleaned with water, and is resistant to insects. These are all properties that are particularly desirable for documentary purposes.

Four commercial operators make a business of laminating documents with the acetate sheeting. Two of these use only heat and pressure; the other two use an adhesive in addition. An adhesive permits lamination at lower temperatures and pressures, but care must be taken that the adhesive is stable and that it will not cause deterioration of the laminated paper.

Because of the high stability of the acetate sheeting, as shown in the Bureau's reports, and its other good qualities, it is also finding considerable use in the form of envelopes for the protection of stamps and other kinds of sheets where lamination is not desirable.

* Communicated by the Director.

DETERMINATION OF PENTOSANS IN PULPS AND PAPERS.

In the Bureau's investigation of the factors affecting the stability of book papers, those chemical properties of pulps and papers were studied which gave promise of yielding useful information. One characteristic was the content of pentosans, a substance or group of substances having the empirical formula $C_5H_8O_4$ and occurring in some fibers in varying amounts.

The most widely used method for the determination of pentosan content at present is that of the Association of Official Agricultural Chemists, which specifies heating the sample with 12 per cent. hydrochloric acid to convert the pentosan to furfural. The furfural is condensed and determined by precipitation as the phloroglucide, the weight of which is used to calculate the pentosan content by an empirical formula.

The method has been variously criticized, especially in that the method of determining furfural is tedious and time-consuming. In an investigation reported in the *Journal of Research* for April (RP1199), Herbert F. Launer and William K. Wilson have adapted, with some changes, the method of determining furfural by oxidation with excess bromate in the cold to the simple and rapid analysis of the distillates from the pentosans. Furthermore, they found that various other methods of distillation did not result in higher pentosan yields. The simple method of distillation was, therefore, retained. By a detailed study of the rate of production of volatile material during the usual distillation procedure and beyond, it was found that the cellulose in the pulps ordinarily used in papermaking gives rise under test conditions to a volatile material, which is determined with the furfural from the pentosans, and thus gives high values for the latter. This was substantiated with an entirely different method, and the value of 0.9 per cent. of pentosan was found by both methods to be applicable as a correction to all of the usual pulps, and to papers made therefrom. Experiments further indicated that cotton materials probably contain only negligible amounts of pentosans and that the values appearing in the literature are due to the reaction products of cellulose.

RP1199 in the April number of the *Journal of Research* should be consulted for a complete account of this work.

PREPARATION AND PROPERTIES OF β-*d*-2-DESOXYGALACTOSE.

The preparation and properties of a new sugar, β-*d*-2-desoxygalactose, are reported in a paper (RP1190) by Horace S. Isbell and William W. Pigman, which will be published in the April number of the *Journal of Research*. The sugar is similar to β-*d*-galactose but differs in that the hydroxyl of the second carbon of galactose is replaced by hydrogen. The sugar was prepared in order to obtain information on the effect of the hydrogen and hydroxyl atoms of the second carbon on the stability of the sugar modifications and the composition of the sugar solutions. The mutarotation of the new sugar shows that it establishes an equilibrium with an alpha pyranose modification and a labile substance.

ACTION OF BAKER'S YEAST ON *d*-TALOSE.

The observation, reported in the current literature, that *d*-talose is not fermentable by baker's yeast is based on studies made with impure talose solutions. Pure crystalline *d*-talose had been prepared in the Bureau's Polarimetry Section, and since the behavior of the sugar towards yeast fermentation is of importance, fermentation studies were made by Horace S. Isbell using pure talose solutions. The results, which are reported in the April *Journal of Research* (RP1191), show that baker's yeast does not ferment *d*–talose and that it cannot be acclimatized for *d*–talose fermentation by growth on glucose or galactose media containing *d*-talose.

INTERNATIONAL TEMPERATURE SCALE AND SOME RELATED PHYSICAL CONSTANTS.

The values in degrees on some recognized scale which are assigned to very high temperatures are obtained from a mathematical formula relating temperature and radiant energy. The actual numbers depend on the value of a certain constant,

c_2, in this formula. This constant can be directly measured or calculated from other atomic constants.

In the past, the value calculated from atomic constants was considered the more accurate and it was therefore used instead of the measured value with which it did not agree. Recent revisions in the values of the atomic constants, particularly in the value of the charge on the electron, have changed the "accepted" values for these constants. The value of c_2, 1.436 cm. deg., calculated from the present values of the atomic constants agrees with the measured values. In the *Journal of Research* for April (RP1189), H. T. Wensel shows that if this value of c_2 is used in place of the value 1.432 cm. deg. defining the International temperature scale, the value for the freezing point of platinum is lowered about 3° C. or 5° F.

The situation in regard to the radiation constants is reviewed primarily for the purpose of selecting the best value of c_2 for use in expressing high temperatures inasmuch as this constant is used in defining the international temperature scale. The experimental values of c_2 and the values derived from other related constants lead to 1.436 cm. deg. as the most probable value.

The analysis of the data lead to a value of $\sigma = 5.70 \times 10^{-5}$ erg sec.$^{-1}$ cm.$^{-2}$ deg.$^{-4}$ for hemispherical radiation. Other radiation constants derived are $\lambda_m T = 2892 \times 10^{-4}$ cm. deg., $c_1 = 3.732 \times 10^{-5}$ erg cm.^{2} sec.$^{-1}$ and the least mechanical equivalent of light = 0.00151 watt ("new" lumen)$^{-1}$.

DENSITY OF GLASSES AS A FUNCTION OF COMPOSITION.

As part of the Bureau's investigation of the variation in density of glass with composition, J. C. Young, F. W. Glaze, and C. A. Faick, under the direction of A. N. Finn, analyzed and determined the densities of a large number of glasses made from varying amounts of powdered quartz crystals, soda ash, and potash which had been melted in platinum crucibles in a platinum resistance furnace.

As reported in the April number of the *Journal of Research* (RP1197), it was found that, between certain compositions, the specific volumes of the glasses vary linearly with composi-

tion and that the limiting compositions correspond to the eutectic compositions determined by studying the crystallizing characteristics of molten glass.

THE SYSTEM CaO-Fe_2O_3-SiO_2.

The system CaO-Fe_2O_3-SiO_2 is the only remaining ternary phase diagram involving major constituents of portland cement which had not been investigated sufficiently to establish boundaries of fields, although Hansen and Bogue (*J. Am. Ceramic Soc.*, **48**, 1261 (1926)) at this Bureau had determined the products of complete crystallization. Accordingly a study of this system was undertaken and 100 compositions have been prepared in the high CaO region of the diagram and including Fe_2O_3 concentrations as high as 70 per cent., and SiO_2 contents up to 45 per cent. A portion of the region investigated is of importance for its bearing on the composition of metallurgical slags.

The results, to date, of quenching determinations and heating curves on these compositions reveal the following salient features of the diagram: (1) a very large field of dicalcium silicate extending to Fe_2O_3 concentrations of nearly 65 per cent.; (2) a very long, narrow field of tricalcium disilicate extending to Fe_2O_3 concentrations of nearly 50 per cent.; (3) a comparatively small field of dicalcium ferrite which is adjacent to the dicalcium silicate field at low SiO_2 concentrations, monocalcium ferrite being adjacent to the dicalcium silicate field at the higher SiO_2 concentrations; (4) a small field of tricalcium silicate in which the liquidus rises steeply with increasing CaO content; and (5) the absence of any ternary compounds in the area studied. The rapidity of crystallization in this system has rendered its study by the quenching method very difficult.

The extent of solid solution of Fe_2O_3 in the high temperature form of dicalcium silicate (beta) has been fixed at 1 per cent. or less. This was determined by holding dicalcium silicate preparations containing small amounts of Fe_2O_3 at appropriate temperatures and examining the preparations microscopically for the presence of glass. Dicalcium silicate

prepared in the ordinary way proved unsuitable for this purpose as the compound on quenching always inverted to the low temperature form. However, if the initial mixture was prepared from a crystalline, hydrated dicalcium silicate, $10CaO \cdot 5SiO_2 \cdot 6H_2O$, this inversion did not occur and the extent of solid solution in beta-dicalcium silicate could be readily determined. The method of preparing the hydrated dicalcium silicate has been described in *J. Research* NBS, **21**, 617 (1938); RP1147.

PERMEABILITY AND STRENGTH OF BRICK-MORTAR JOINTS.

The Bureau's Building Materials and Structures Report BMS7, "Water Permeability of Masonry Walls," gives the results of a few tests indicating that the water permeability of brick masonry walls is affected by the rate of absorption of the brick at the time they are laid. More recent tests confirm this indication. Data available in the literature indicate that the tensile strength of brick-mortar joints also is affected by the rate of absoprtion of brick.

A convenient measure of the rate of absorption of brick, or penetrability, is the amount of water, in grams, absorbed by a brick when its flat face is immersed to a depth of $\frac{1}{8}$ in. in water for one minute. The penetrability of clay building bricks, when dry, ranges from less than 5 to more than 150 grams. It may be reduced to any desired amount by wetting the bricks prior to use. The practical significance of the penetrability lies in the fact that it serves as a measure of the rate at which a brick tends to absorb water when placed in contact with mortar. When bricks of high penetrability are used they tend to cause a rapid stiffening of the mortar resulting from the absorption of part of the mixing water.

In tests of the water permeability of brick masonry walls and of the tensile strength of brick-mortar joints, it has been found that the permeability of the masonry and the tensile strength of the joints is greater the lower the penetrability of the brick, irrespective of whether the bricks are naturally of extremely low absorption or whether their penetrability has been reduced by wetting prior to setting. When the penetrability exceeded 50 grams the strength of the bond was low and

erratic. Similarly, the resistance to rain penetration of masonry walls was poor when the penetrability exceeded 50 grams per brick, erratic but generally poor when the penetrability was between 20 and 50 grams, and usually good or excellent when the penetrability was less than 20 grams.

The results point strongly to the desirability of wetting absorbent bricks before laying. The only limit to the amount of wetting seems to be the practical one that if too wet the bricks will tend to slide on the joints or to "float." The data indicate, therefore, that the general rule to be followed is to have the bricks as wet as they can be laid conveniently.

PROPERTIES OF FIBER BUILDING BOARDS.

Data on the properties of fiber building boards of current manufacture are contained in Building Materials and Structures Report BMS 13, copies of which are obtainable from the Superintendent of Documents, Government Printing Office, at 10 cents each. The boards tested included the ordinary fiber insulating boards, interior-finish insulating boards, and a new class of commercial fiber boards known as insulation sheathing. The boards in all three classes are of low density to provide low thermal conductivity, which means good heat insulation. Low density is obtained by the interlacing of the fibers to form millions of minute air cells. These air cells provide the real barrier to the passage of heat. The wall-finish boards and the ordinary insulating boards differed only in that the former had a special finish on one side. The new type of sheathing boards are designed for use as siding. They were nearly an inch in thickness and have asphalt treatments for protection against infiltration of water and air. The stability of the boards is the subject of a study now in progress.

Preliminary Report on the New Jersey Earthquakes of Aug. 1938.—The report by the Coast and Geodetic Survey on the series of moderate earthquake disturbances which shook the central portion of New Jersey and surrounding territory on the night of Aug. 22–3 has been made possible through the generous coöperation of postmasters in the affected area who furnished information requested by canvass, and by the coöperation of volunteer observers of the U. S. Weather Bureau and other correspondents. The center of the disturbance has been located in the western corner of Monmouth County, N. J., about 15 miles southeast of Trenton. This location is based on a study of instrumental data obtained from the seismograph stations of the Franklin Institute, Phila., and Fordham University, N. Y. It is verified by the results of the survey of the affected territory for information concerning the distribution of intensity. The first strong shock occurred at 10.36 P.M. E.S.T. on Aug. 22. Another strong shock occurred in practically the same place at 2.03 A.M. on the 23d, and still another, but weaker, at 6.11 A.M. Weak shocks occurred at approximately 12.10 and 12.20, and 3.05 A.M. On Aug. 27th at 5.36 P.M. two sharp shocks, about a minute apart, were again felt over a rather wide area reaching almost as great an intensity in the central area as the strongest of the series. The cause of these shocks is more or less a matter of conjecture. It may be significant that the area of maximum intensity near Trenton and the area of high intensity near Philadelphia both lie along a zone where the heavier formations associated with the Appalachian Mountain system give way to the lighter formations which characterize the southern portion of New Jersey. This zone which extends in a northeasterly direction into the Hudson River Valley apparently defines a line along which occasional adjustments may be expected on the basis of past seismic history. The elongation of the area of high intensity along this zone also tends to mark it as a zone of seismic slipping.

R. H. O.

THE FRANKLIN INSTITUTE

STATED MONTHLY MEETING, APRIL 19, 1939.

The regular monthly meeting of The Franklin Institute was called to order by Mr. Philip C. Staples, President, at 8:30 o'clock P.M.

The minutes of the previous meeting were approved as published in the current issue of the JOURNAL.

The additions to membership since the last report were as follows:

Active	11
Associate	10
Student	61
Total	82

The Board of Managers had nominated Dr. Edwin Powell Hubble, the recipient of the Franklin Medal, for Honorary Membership. No objection being offered, it was so ordered.

The Chairman then introduced Dr. W. F. G. Swann, Director of the Bartol Research Foundation of The Franklin Institute, who presented his Annual Report * of the activities of the Laboratory. The report covered the significant investigations which have been in progress during the past year and in particular it dealt with researches in nuclear physics and cosmic rays. Accounts were given of phenomena concerning high altitude investigations; the properties of the newly discovered cosmic-ray particle—the mesotron; researches which have been made seeking evidence for the existence of neutrons in cosmic rays; and theories evolved in harmonization of the phenomena. Several developments in instrumental technique were also described. The lecture was illustrated by experiments and lantern slides.

The Chairman conveyed the thanks of the meeting to Dr. Swann.

Adjourned.

HENRY BUTLER ALLEN,
Secretary.

* To be published in an early issue of the JOURNAL.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, April 12, 1939.)

HALL OF THE COMMITTEE,
PHILADELPHIA, APRIL 12, 1939.

MR. WILLIAM G. ELLIS *in the Chair.*

The following reports were presented for final action:

No. 3061: Electrolytic Reduction of Sugars.

This report recommended the award of a Howard N. Potts Medal to H. Jermain Creighton, of Swarthmore, Pennsylvania, "In consideration of his distinguished work in developing a process for the electrolytic reduction of simple sugars on a large scale."

No. 3062: Work of Sir Charles Boys.

This report recommended the award of an Elliott Cresson Medal to Sir Charles Vernon Boys, of London, England, "In recognition of his scientific achievements which have furnished scientists with new and remarkably precise methods for making measurements in gravitation, in sound, in heat, in radiation, in current and static electricity so refined as to render possible the weighing of the earth, the determination of the radiant energy coming to us from the stars, the photography of the speeding bullet, and the immobilization, even, of the lightning flash itself."

No. 3068: Henderson Medal.

This report recommended the award of a George R. Henderson Medal to Ralph Budd, of Chicago, Illinois, "In consideration of his outstanding contributions to the advancement of railroads in this country and abroad, through a career characterized by vision, skill, thorough engineering knowledge, and keen executive ability."

At the February 8, 1939, Stated Meeting, the following report was presented for final action:

No. 3042: Centrifugal Impregnator.

This report recommended the award of a John Price Wetherill Medal to William A. Hyde, of Worcester, Massachusetts, "In consideration of his discovery and development of a process for impregnation of objects which has materially reduced the time necessary for impregnation as compared with the prior art."

This recommendation was advertised in the March issue of the Journal of The Franklin Institute, and the advertisement has expired without the receipt of any objection.

JOHN FRAZER,
Secretary to the Committee.

LIBRARY NOTES.

The Committee on Library desires to add to the collections of the Institute any technical writings of members who have had occasion to publish such material.

Literary contributions from author-members will be gratefully acknowledged, properly inscribed and noted in the Journal of the Institute.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The cost for a print 9 × 14 inches is forty cents.

The library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from nine o'clock A.M. until five o'clock P.M., Wednesdays and Thursdays two until ten o'clock P.M. 836 readers made use of the facilities during the twenty-seven days of March.

RECENT ADDITIONS.

CHEMISTRY AND CHEMICAL TECHNOLOGY.

CURWEN, M. D., ed. Chemistry in Commerce. Four volumes. No date.

Electrochemical Society. Transactions, October 1938. Volume 74. 1939.

HEDVALL, J. ARVID. Reaktionsfähigkeit fester Stoffe. 1938.

IRWIN, FREDERICK C., AND G. RAY SHERWOOD. General and Inorganic Chemistry. 1939.

LAMIRAND, J., AND CH. BRUNOLD. Chimie Générale et Métalloïdes. Troisième Edition. 1935. Cours de Chimie. Première Partie.

LAMIRAND, J., AND H. PARISELLE. Chimie Organique. Deuxième Édition, Revue. 1938. Cours de Chimie. Troisième Partie.

LIVINGSTON, ROBERT. Physico-Chemical Experiments. 1939.

MACINNES, DUNCAN A. The Principles of Electrochemistry. 1939.

SCOTT, WILFRED W. Standard Methods of Chemical Analysis. Fifth Edition. Edited by N. Howell Furman. Two volumes. 1939.

SHERWOOD, THOMAS K., AND CHARLES E. REED. Applied Mathematics in Chemical Engineering. First Edition. 1939.

SUTERMEISTER, EDWIN, AND FREDERICK L. BROWNE. Casein and its Industrial Applications. Second Edition. 1939.

WHITTAKER, C. M., AND C. C. WILCOCK. Dyeing with Coal Tar Dyestuffs. Third Edition. 1939.

WINTON, ANDREW L., AND KATE BARBER WINTON. The Structure and Composition of Foods. Volume 4. 1939.

DIRECTORIES.

New Jersey Bureau of Statistics and Records. Industrial Directory of New Jersey. 1938.

ELECTRICITY AND ELECTRICAL ENGINEERING.

GHIRARDI, ALFRED A. Radio Trouble-Shooter's Handbook. First Edition. 1939.

KRON, GABRIEL. Tensor Analysis of Networks. 1939.

MYERS, L. M. Electron Optics. 1939.

TYNDALL, A. M. The Mobility of Positive Ions in Gases. 1938.

ENGINEERING.

Manchester Association of Engineers. Some Episodes in the Manchester Association of Engineers. 1938.

GEOLOGY.

DU TOIT, ALEX. L. Our Wandering Continents. 1937.

PICKWELL, GAYLE. Deserts. 1939.

INVENTIONS AND DISCOVERIES.

MALONEY, JOHN A., AND ASSOCIATES. Great Inventors and their Inventions.

MANUFACTURES.

ERNST, HANS, AND OTHERS. Machining of Metals. 1938.

TURNER, T. HENRY, AND N. F. BUDGEN. Metal Spraying. Second Edition, by E. C. Rollason. 1939.

MATHEMATICS.

HAGUE, B., An Introduction to Vector Analysis for Physicists and Engineers. 1939.

ROOM, T. G. The Geometry of Determinantal Loci. 1938.

SLOANE, T. O'CONOR, J. E. THOMPSON, AND H. E. LICKS. Speed and Fun with Figures. 1939.

METALLURGY.

BULLENS, D. K. Steel and its Heat Treatment. Volume 2: Engineering and Special Purpose Steels. Fourth Edition. 1939.

KEHL, GEORGE L. The Principles of Metallographic Laboratory Practice. 1939.

U. S. Steel Corporation. Chemists of the Subsidiary Companies. Sampling and Analysis of Carbon and Alloy Steels. 1938.

MINING AND MINERALOGY.

ENGLISH, GEORGE LETCHWORTH. Descriptive Lists of New Minerals 1892–1938. First Edition. 1939.

Mineral Industry: its Statistics, Technology and Trade during 1937. Volume 46. 1938.

NAVAL ARCHITECTURE AND NAVIGATION.

ANGAS, W. MACK. Rivalry on the Atlantic. 1939.

PHARMACY.

KAY, LILIAN A. The Microscopical Study of Drugs. 1938.

PHOTOGRAPHY.

Jahresbericht der Photographie, Kinematographie und Reproduktionstechnik für das Jahr 1937. 1938.

MERTLE, J. S. Photolithography and Offset Printing. 1937.

PHYSICS.

Grimsehls Lehrbuch der Physik. Neubearbeitet von Prof. Dr. R. Tomaschek. Erster Band. Zehnte Auflage. 1938.

Indian Academy of Sciences. Sir Chandrasekhara Venkata Raman 1888—7th November—1938. Jubilee Volume of Original Papers Presented in Commemoration of his Fiftieth Birthday. 1938.

STEWART, OSCAR M. Physics. Third Edition. 1939.

REFRIGERATION.

American Society of Refrigerating Engineers. Refrigerating Data Book. Fourth Edition. Volume 1. 1939.

SCIENTIFIC ESSAYS.

FEZANDIÉ, CLEMENT. Through the Earth. 1898.

FINANCIAL STATEMENTS

as at December 31, 1938.

April 14, 1939.

To the Board of Managers of
The Franklin Institute,
Philadelphia.

We have made an examination of the balance sheet of THE FRANKLIN INSTITUTE as at December 31, 1938, and of the statements of income and expenses and general fund for the year ended that date. In connection therewith, we examined or tested accounting records of the Institute and other supporting evidence, and obtained information and explanations from officers and employees of the Institute; we also made a general review of the accounting methods, but we did not make a detailed audit of the accounts.

The income as stated in the accompanying statement of income and expenses is as recorded in the books of account. The expenses were supported by proper vouchers. Income from fund accounts was verified by us. The buildings and equipment and investments are stated at cost or appraised values at date of acquisition, and do not purport to represent present day values.

On the basis stated above, the annexed financial statements, in our opinion, fairly present, in accordance with accepted principles of accounting consistently maintained by the Institute during the year under review, the financial position of The Franklin Institute at December 31, 1938, and the results of its operations for the year ended that date, after excluding the accounts of The Bartol Research Foundation of The Franklin Institute, and of The Biochemical Research Foundation of The Franklin Institute, on which separate reports were submitted.

(Signed) LYBRAND, ROSS BROS. & MONTGOMERY.

THE FRANKLIN INSTITUTE.

BALANCE SHEET, DECEMBER 31, 1938.

Assets.

Cash in banks and on hand (not including cash in endowment trust funds)		$35,222.93
Accounts receivable for memberships, subscriptions, etc.		14,451.49
Materials and supplies, at cost		11,781.83
Buildings and equipment, at cost, not including exhibits loaned and donated		4,070,905.04
Deferred expenses		3,035.65
		$4,135,396.94
Endowment fund investments, cash etc.:		
Investments, at cost or appraised values at the dates of acquisition	$2,191,537.85	
Deposit in closed bank, less reserve	67,500.00	
Cash:		
Uninvested principal	64,985.36	
Unexpended income	7,371.81	
		2,331,395.02
		$6,466,791.96

Liabilities.

Accounts payable		$13,750.50
Amounts received for life memberships not transferred to General Endowment Fund		1,300.00
Deferred income from dues, etc.		36,530.51
Contributed funds used for buildings, equipment, etc., as annexed		3,922,550.89
Stock outstanding:		
First class	$12,940.00	
Second class	70.00	
		13,010.00
General Fund account, as annexed		148,255.04
Endowment funds:		
Principal, as annexed	$2,324,023.21	
Unexpended income	7,371.81	
		2,331,395.02
		$6,466,791.96

GENERAL FUND ACCOUNT

for the year ended December 31, 1938.

Balance, January 1, 1938		$123,041.20
Add amount withdrawn from the Museum Reserve Fund Principal for:		
Payments on statue	$28,000.00	
Other capital and operating expenditures	85,000.00	
		113,000.00
		$236,041.20
Deduct:		
Claim for taxes withheld in settlement covering sale of premises at Nos. 13–15–17 South Seventh Street, Philadelphia, denied by Receiver of Taxes	$5,407.84	
Miscellaneous accounts receivable arising in prior years considered uncollectable	162.02	
Deficit for the year ended December 31, 1938	82,216.30	
		87,786.16
Balance, December 31, 1938		$148,255.04

INCOME AND EXPENSE ACCOUNT

for the year ended December 31, 1938.

Income.

Dues (prorated portion of annual dues), initiation fees, etc.			$22,082.92
Less dues uncollected, prior years			405.00
			$21,677.92
Admissions to museum and planetarium, sale of guide books, etc.			77,117.45
Subscriptions to publications and advertising			7,704.29
Donations:			
Dedication exercises		$23,824.86	
Friends of Franklin	$70,452.39		
Less expenses	8,554.87	61,897.52	
			85,722.38
Administrative charges for services rendered The Bartol Research Foundation and The Biochemical Research Foundation			10,000.00
Miscellaneous income, net of expenses incident thereto			2,208.89
			$204,430.93
Income from trust fund investments, etc.			70,579.11
Total Income (*Carried Forward*)			$275,010.04

INCOME AND EXPENSE ACCOUNT (*Continued*)

Total Income (*Brought Forward*)		$275,010.04

Expenses.

	Salaries and Wages.	Other.	
Building operation and maintenance	$39,580.41	$34,912.75	
Library	14,387.20	5,505.25	
Meetings	—	2,167.20	
Office and general	50,012.65	12,148.17	
Machine shop	732.78	98.41	
Museum	104,939.53*	45,082.04*	
Publications	—	14,314.34	
Science and The Arts	3,762.59	219.33	
Traveling expense	—	745.31	
Annuities	3,125.00	—	
Membership badges, certificates, etc.	—	1,063.67	
Dedication exercises	3,973.57	20,456.14	
	$220,513.73	$136,712.61	
			357,226.34
Deficit for the year			$82,216.30

* These amounts are stated net of donations of $15,669.31 of which $8,930.36 was applied against salaries and wages and $6,738.95 against other expenses.

SUMMARY OF CHANGES IN ENDOWMENT FUNDS

for the year ended December 31, 1938.

Balances, January 1, 1938			$2,464,916.16
Add:			
Profit on sale or call of investments		$19,075.69*	
Amounts received from:			
Estate of George S. Pepper	$1,886.13		
Estate of Craig D. Ritchie	1,944.86	3,830.99	
Miscellaneous		6.72	
			22,913.40
Total (*Carried Forward*)			$2,487,829.56

* Includes $1,393.08 profit on transfer of Museum Reserve Fund investments at market values to Investments of other funds.

SUMMARY OF CHANGES IN ENDOWMENT FUNDS (*Continued*)

Total (*Brought Forward*)			$2,487,829.56
Deduct:			
Losses on sale of investments		$31,227.70†	
Amounts withdrawn from the Museum Reserve Fund:			
For payments on statue of Benjamin Franklin	$28,000.00		
For other capital and operating expenditures	85,000.00		
Dividend on closed bank account transferred to Benjamin Franklin Memorial, Inc.	17,500.00	130,500.00	
Amounts received as a distribution from principal of the John G. Meigs Bequest, credited to John G. Meigs Memorial Fund (income account)		2,078.65	
			163,806.35
Balances, December 31, 1938			$2,324,023.21

† Includes $1,652.23 loss on transfer of Museum Reserve Fund investments at market values, to investments of other funds.

CONTRIBUTED FUNDS RECEIVED FROM BENJAMIN FRANKLIN MEMORIAL, INC.,

for the year ended December 31, 1938.

Balance, January 1, 1938			$3,902,492.98
Add:			
Pledges collected		$2,557.91	
Dividend received on deposit account with Franklin Trust Company (closed bank), restored to account		17,500.00	
			20,057.91
Balance, December 31, 1938			$3,922,550.89

THE BARTOL RESEARCH FOUNDATION OF THE FRANKLIN INSTITUTE.

CERTIFIED FINANCIAL STATEMENTS

as at September 30, 1938.

December 20, 1938.

To the Board of Directors of
The Franklin Institute,
Philadelphia.

We have made an examination of the balance sheet of THE BARTOL RESEARCH FOUNDATION OF THE FRANKLIN INSTITUTE as at September 30, 1938, and of the statement of income and expenses for the year ended that date. In connection therewith, we examined or tested accounting records of the Foundation and other supporting evidence and obtained information and explanations from officers and employees of The Franklin Institute; we also made a general review of the operating and fund accounts for the period, but we did not make a detailed audit of the transactions. The building, which is located on the property of Swarthmore College, and the equipment are stated at cost and the investments at cost or appraisal values at dates of acquisition and do not purport to represent present day values. The cost of books and scientific literature is charged to expense.

Subject to the foregoing, the accompanying balance sheet and related statement of income and expenses, in our opinion fairly present, in accordance with accepted principles of accounting consistently maintained by the Foundation during the year under review, its financial position at September 30, 1938, and the results of its operations for the year ended that date.

(Signed) LYBRAND, ROSS BROS. & MONTGOMERY.

THE BARTOL RESEARCH FOUNDATION OF THE FRANKLIN INSTITUTE.

BALANCE SHEET, SEPTEMBER 30, 1938.

Assets

Cash:			
In bank and on hand, general fund		$3,936.83	
Unexpended income:			
Henry W. Bartol Bequest	$5,185.99		
Investment of unexpended income fund	7,455.41	12,641.40	
			$16,578.23
Investments of unexpended income fund, at cost			217,231.75
Account receivable			3.37
Building and equipment, at cost:			
Laboratory building		$226,591.05	
Furniture and fixtures		14,874.93	
Equipment:			
Laboratory	$83,485.14		
Less allowance for depreciation	16,413.13	67,072.01	
Machine shop	$16,849.39		
Less allowance for depreciation	2,168.51	14,680.88	
Building		375.00	
Office		3,013.47	
Library		1,057.65	
			327,664.99
			$561,478.34

Investments (at cost or appraised values at dates of acquisition) and cash of principal of trust funds:

	Investments	Cash	
Henry W. Bartol Bequest	$1,316,369.45	$6,094.35	
Pension fund	75,190.94	991.38	
	$1,391,560.39	$7,085.73	
			1,398,646.12
ASSETS (*Carried Forward*)			$1,960,124.46

ASSETS, SEPTEMBER 30, 1938 (*Continued*)

Total Balance Sheet (*Brought Forward*)			$1,960,124.46
	Collateral Trusts.		
Cash:			
Uninvested principal	$3,899.67		
Undistributed income	2,787.29	$6,686.96	
Investments:			
Helene Bonar Bartol Trust	$309,060.15		
Lucy Cheyney Farr Trust	154,808.09		
Katherine de Monclos Trust	155,174.93	619,043.17	
			625,730.13
			$2,585,854.59
	Liabilities		
Accounts payable			$1,979.88
General Fund account:			
Balance, October 1, 1937		$554,274.69	
Reimbursement for labor and material furnished The Biochemical Research Foundation of The Franklin Institute, applicable to prior year		1,236.67	
		$555,511.36	
Excess of income over expenses for the year ended September 30, 1938, as annexed		3,987.10	
			559,498.46
			$561,478.34
Endowment Funds:			
Henry W. Bartol Bequest		$1,322,463.80	
Pension fund		76,182.32	
			1,398,646.12
			$1,960,124.46
	Collateral Trusts.		
Principal:			
Helene Bonar Bartol Trust	$311,264.43		
Lucy Cheyney Farr Trust	155,905.88		
Katherine de Monclos Trust	155,772.53	$622,942.84	
Undistributed income		2,787.29	
			625,730.13
			$2,585,854.59

INCOME AND EXPENSE ACCOUNT

for the year ended September 30, 1938.

Income

Income received from investments		$61,427.99

Expenses.

	Salaries and Wages.	Other.	
Building operation and maintenance	$1,072.00	$2,760.65	
Library	—	755.45	
Administrative and general	15,100.00	2,077.10	
Laboratory and machine shop	22,187.58	3,134.17*	
Publications, reprints	—	225.28	
Traveling expense	—	102.95	
Administrative charge, The Franklin Institute	—	5,000.00	
Investment counsel charge	—	458.20	
	$38,359.58	$14,513.80	
			52,873.38
			$8,554.61
Other expense, Loss, net of profits, on the sale of income investments			4,567.51
Excess of income over expenses for the year, transferred to General Fund Account			$3,987.10

* Includes provision of $949.20 for depreciation of laboratory equipment.

THE BIOCHEMICAL RESEARCH FOUNDATION OF THE FRANKLIN INSTITUTE.

CERTIFIED FINANCIAL STATEMENTS

as at August 31, 1938.

December 20, 1938.

To the Board of Managers of
The Franklin Institute,
Philadelphia.

We have made an examination of the balance sheet of THE BIOCHEMICAL RESEARCH FOUNDATION OF THE FRANKLIN

INSTITUTE as at August 31, 1938, and of the statements of expenses and income and the general fund account for the year ended that date. In connection therewith, we examined or tested the accounting records of the Foundation and other supporting evidence and obtained information and explanations from officers and employees of The Franklin Institute; we also made a general review of the accounting methods and of the operating and fund accounts for the period, but we did not make a detailed audit of the transactions. The research apparatus and other equipment are stated at their appraised values on July 1, 1935, with additions since that date at cost; the investments are stated at appraised values at dates of acquisition.

Subject to the foregoing, the accompanying balance sheet and the related statements of expenses and income and general fund account, in our opinion, fairly present, in accordance with accepted principles of accounting consistently maintained by the Foundation during the period under review, its financial position at August 31, 1938, and the results of its operations for the year ended that date.

(Signed) LYBRAND, ROSS BROS. & MONTGOMERY.

THE BIOCHEMICAL RESEARCH FOUNDATION OF THE FRANKLIN INSTITUTE.

BALANCE SHEET, AUGUST 31, 1938.

Assets.

Cash:			
In general checking account and on hand		$9,486.95	
Unexpended cash of funds available for current operations:			
Biochemical Research Foundation	$17,435.10		
Emma M. Schaible Bequest	1,207.69	18,642.79	
Unexpended cash of funds available for bonuses to employees (see contra)		16,389.80	
			$44,519.54
Investments, at appraised values at dates of acquisition:			
Biochemical Research Foundation		$315,620.00	
Emma M. Schaible Bequest		1,760.50	
ASSETS (*Carried Forward*)			$317,380.50

ASSETS, AUGUST 31, 1938 (*Continued*)

Total Balance Sheet (*Brought Forward*)		$317,380.50
Equipment, at appraised value July 1, 1935, with subsequent additions at cost:		
Research apparatus and equipment	$137,791.20	
Improvements to leased property	11,461.75	
Cyclotron experiment	22,350.63	
Library books	3,942.76	
		175,546.34
		$537,446.38

Liabilities

Accounts payable	$1,741.89
Unexpended income from investments, available for bonuses to employees (see contra)	16,389.80
Deferred income, donation applicable to Contingent Fund expense during the year ending August 31, 1939	2,500.00
General Fund account, as annexed	516,814.69
	$537,446.38

STATEMENT OF EXPENSES AND INCOME

for the year ended August 31, 1938.

Expenses.

	Salaries and Wages.	Other.	
Building operation and maintenance		$11,647.82	
Administrative and general	$31,118.04	3,192.39	
Laboratory	105,597.20	13,582.50	
Cyclotron	2,990.55	3,518.20	
Publications and printing	—	881.13	
Dues and subscriptions	—	1,181.43	
Traveling and entertainment	—	2,034.37	
Administrative charge, The Franklin Institute	—	5,000.00	
Miscellaneous	—	309.55	
	$139,705.79	$41,347.39	$181,053.18
Advance to the Hendry-Connell Research Foundation, Ltd., under agreement dated December 6, 1935			25,000.00
Loss on sales of investments			28,141.21
Statement of Expenses and Income (*Carried Forward*)			$234,194.39

STATEMENT OF EXPENSES AND INCOME (*Continued*)

Statement of Expenses and Income (*Brought Forward*)		$234,194.39
Income.		
Donation	$2,500.00	
Income received from investments, Emma M. Schaible Bequest	165.00	
Royalties received on sales of book, "Cancer and Diet"	235.43	
		2,900.43
Excess of expenses over income for the period, transferred to General Fund account		$231,293.96

GENERAL FUND ACCOUNT

for the year ended August 31, 1938.

Balance, September 1, 1937		$381,110.65
Add:		
Appraised value of 200 shares Christiana Securities Company common stock, received as donation		367,000.00
		$748,110.65
Less:		
Nominal value of stock of The Hendry-Connell Research Foundation, Ltd., charged off due to termination of agreement as of January 19, 1938	$2.00	
Excess of expenses over income available for purposes of the Foundation, for the year ended August 31, 1938, as annexed	231,293.96	231,295.96
Balance, August 31, 1938		$516,814.69

NOTES FROM THE BARTOL RESEARCH FOUNDATION.

DETERMINATION OF THE SHOWER PRODUCING EFFICIENCY OF COSMIC-RAY PARTICLES.*

BY

W. E. RAMSEY AND W. E. DANFORTH.

The approximate agreement of the Bethe-Heitler theory with observations on electrons [1] adds to the importance of determining the shower producing capacity of those rays which are recorded in a conventional counter telescope. For this purpose we counted coincidences between two layers of very thin walled Geiger counters operating at a pressure so low that the probability of a single ray setting off a counter is very much less than unity. We then utilized the relatively high efficiency of these counters for multiple rays in detecting electron showers originating above them. Such an arrangement differs from the usual counter shower device in that the probability of recording a shower is independent of the angular spread of the shower electrons. This fact simplifies the assignment of an upper limit to the total number of showers arising in a specimen as a result of the passage through it of a known number of single rays.

If we count coincidences between the counter groups A and B (Fig. 1) and if P is the probability that a single ray will set off any one counter in A or B, then P^2 is the probability that a single ray in passing through A and B will be recorded as a coincidence. However if n rays pass through A and B simultaneously, the probability of their being recorded is $\{1 - (1 - P)^n\}^2$. If we take the coincidence counting rate of A and B at high pressure, where the counters are practically 100 per cent. efficient, we obtain the total number of rays passing through the counter surfaces. The same measurement taken at reduced pressure gives a determination of P for

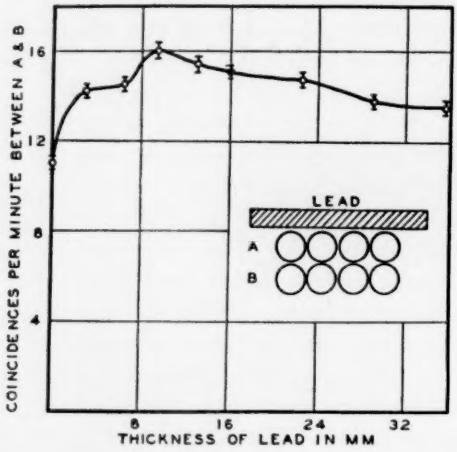
* Reprinted from the *Physical Review*, **51**, 1105 (1937).

[1] C. D. Anderson and S. H. Neddermeyer, *Phys. Rev.*, **50**, 263 (1936).

that pressure, and the corresponding efficiency of the coincidence set for a shower of any size. These measurements were taken in open air, and multiple rays passing through A and B in the absence of lead, or single rays which pass through more than one counter of A or B, are not sufficiently numerous to affect seriously the calculation of P.

The results shown in the curve were taken with 2 cm. of hydrogen in the counters. At this pressure the coincident

FIG. 1.



Arrangement of counters and number of coincident counts per minute with hydrogen at 2 cm. pressure in the counters.

efficiency of A and B is 6.3 per cent. for single rays, hence 19 per cent. for two rays, 34 per cent. for three rays, 46 per cent. for four rays, and so on. Employing a relation obtained by C. G. Montgomery and D. D. Montgomery [2] for the frequency of occurrence of showers as a function of shower size we conclude that the average number of rays in a shower from 1.5 cm. of lead is four. Assuming this figure for n, and utilizing the maximum increase indicated in our curve, it follows that not more than 6 per cent. of all the ionizing rays passing through A and B produce a shower in lead of this thickness. If we take $n = 2$ we obtain an absolute maximum

[2] C. G. Montgomery and D. D. Montgomery, *Phys. Rev.*, **50**, 490 (1936).

of 15 per cent. This figure is obviously too high as $n = 2$ corresponds to the smallest shower that can arise in the lead.

If the incoming rays are electrons, having the energy distribution given by Blackett,[3] and producing showers according to the Bethe-Heitler theory, we should have obtained an increase in counting rate with lead not less than ten times that actually observed. If, as seems to be the case, the theory is correct for electrons we are led to conclude that the bulk of those rays which record themselves in counter sets at sea level is not electrons. Rossi,[4] J. C. Street, R. W. Woodward and E. C. Stevenson [5] have concluded that the penetrating component of the radiation is not an electron. Since the penetrating component is dominant at sea level our results are in agreement with this conclusion.

CONSERVATION OF ENERGY IN THE DISINTEGRATION OF Li^8.*

BY

L. H. RUMBAUGH, R. B. ROBERTS, AND L. R. HAFSTAD.†

Some time ago two of us reported an investigation with isotopic targets of the energy balance in the formation of radio-lithium by the process $Li^7 + D^2 \rightarrow Li^8 + H^1$, and showed that the emission of a beta-particle must be followed by yet another reaction to absorb an excess of energy which amounts to more than 3 Mev., the predicted processes involving the emission of alpha-particles, gamma-rays, or both.[1] The alpha-particles have been found by Lewis, Burcham, and Chang [2] who report that, though most of the alpha-particles have ranges of less than 1.5 cm. of air, about 0.3 per cent. of the total extend to 5 cm. They suggest that the alpha-

[3] P. M. S. Blackett, *Proc. Roy. Soc.*, **159**, 1 (1937).

[4] B. Rossi, *Zeits. f. Physik*, **82**, 151 (1933).

[5] J. C. Street, R. H. Woodward, and E. C. Stevenson, *Phys. Rev.*, **47**, 891 (1935). Also J. C. Street, E. C. Stevenson, *Phys. Rev.*, **51**, 1005 (1937).

* Reprinted from the *Physical Review*, **51**, 1106 (1937).

† Drs. Roberts and Hafstad are members of the staff of the Department of Terrestrial Magnetism, Carnegie Institution of Washington, Washington, D. C.

[1] Rumbaugh and Hafstad, *Phys. Rev.*, **50**, 681 (1936).

[2] Lewis, Burcham, and Chang, *Nature*, **139**, 24 (1937).

particles originate in excited Be^8 formed "either in a number of discrete energy states or in a continuous distribution."

We have continued our investigations of the formation and disintegration of radio-lithium with the following results.

(1) The initial reaction, $Li^7 + D^2 = Li^8 + H^1 + Q$, seems the only possible one fitting the experimental facts.

(2) The energy balance, Q_1, for the formation of Li^8 must be greater than $-$ 360 kv. since radio-lithium was formed by deuterons of energies as low as 360 kv.

(3) Q_1 is not greater than + 260 kv. since, using 860-kv. deuterons, no proton group of range exceeding 1.7 cm. was observed with even a fraction of the intensity necessary for correlation with the amount of radio-lithium produced.

(4) No gamma-radiation was observed in the formation or disintegration of Li^8 although the gamma-radiation from much thinner carbon-targets was readily detected.

(5) The ratio of alpha-particles of ranges exceeding 6 mm. to beta-particles counted under the same experimental conditions is very nearly unity. However, corrections for alpha-particles of shorter range and for the geometry of the beta-particle counter would tend to increase this ratio.

(6) The excitation curves for the delayed alpha- and beta-particles as shown in Fig. 1 are identical and markedly different from eight other carefully measured lithium excitation curves.

(7) The disintegration of Li^8 produces alpha-particles with a half-period of 0.85 $\pm$ 0.1 sec. and beta-particles with a half-period of 0.90 $\pm$ 0.1 sec., indicating that the decay periods are identical.

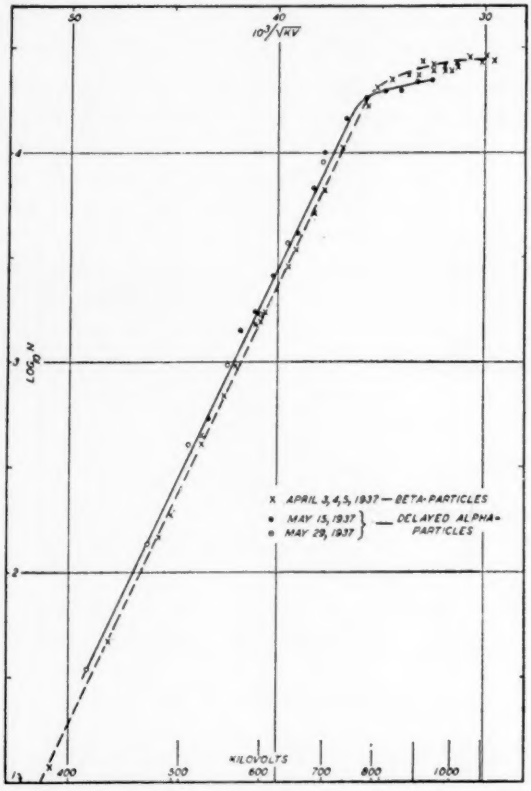
(8) We have been unable to observe the instantaneous emission of low energy alpha-particles which might be produced by bombardment of Li^6 with deuterons or Li^7 with protons. These might be expected if the delayed alpha-particles arise from excited states of Be^8.

(9) The number range curve for the delayed alpha-particles shows a smooth continuous distribution falling rapidly with increasing range, and decreasing by a factor of 800 between 0.6 and 5.2 cm.

In view of the experimental results (4), (5), (6), (7), and (8) above, it is highly probable that Li^8 always disintegrates into two alpha-particles and a beta-particle. Combining this

fact with (9) and the known form of the energy-distribution curve for beta-rays, it follows that, for any given energy interval, both conservation of energy and of electric charge

FIG. 1.



Yield of delayed particles from $Li^7 + D^2$.

cannot be satisfied simultaneously by considering charged particles alone.

This difficulty is avoided by including an uncharged body (neutrino hypothesis) in the four-body disintegration

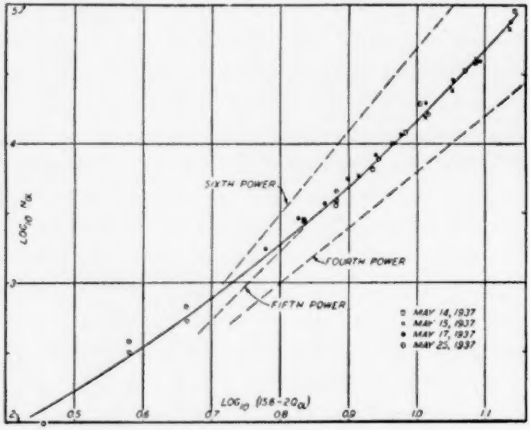
$$_3Li^8 \rightarrow {}_2He^4 + {}_2He^4 + {}_{-1}\epsilon + {}_0\nu + Q_2.$$

The disintegration energy Q_2 is divided between Q_L, the energy of the light particles, and Q_H, the energy of the heavy particles. From conservation of momentum Q_H is very nearly equal to $2Q_\alpha$, twice the energy of a single alpha-particle. Using the Bainbridge-Jordan [3] mass scale and $Q_1 \sim 0.2$ Mev., according to (2) and (3) above, $Q_2 \sim 15.6$ Mev., whence

$$Q_L = (15.6 - 2Q_\alpha).$$

Since, by conservation of charge, the transition probability for a beta-neutrino energy, Q_L, will be proportional to the

FIG. 2.



Energy diagram for Li^8 disintegration.

number of alpha-particles, N_α, observed with energies Q_α, the relation between this transition probability and Q_L may be obtained by plotting N_α against Q_L as in Fig. 2.

It is interesting to note that *the application of the neutrino hypothesis, conservation of energy, and charge to our alpha-particle data results in a Sargent diagram.* In fact, this diagram is essentially an idealized Sargent diagram because it represents a *continuum* in Q_L for which initial and final nuclei are identical.

[3] Bainbridge and Jordan, *Phys. Rev.*, **51**, 384 (1937).

We are greatly indebted to Professors Gamow and Teller for discussion of the theory of this disintegration. They have pointed out that considerations based on the Fermi theory predict a number of alpha-particles proportional to $(Q - 2E_\alpha)^5(E_\alpha)^n$ where n is $\frac{1}{2}$ or $\frac{3}{2}$ depending on the angular momentum of the alpha-particles. Our observations agree satisfactorily with the fifth power (Fig. 2), although further study of the low energy alpha-particles will be necessary to decide whether n should be $\frac{1}{2}$ or $\frac{3}{2}$. The Konopinsky-Uhlenbeck theory predicts a seventh power dependence which clearly is not in agreement with our observations.

The above interpretation at once suggests additional experiments to establish whether or not the conservation of momentum is also satisfied by the neutrino hypothesis in the disintegration of Li^8. Preparations for such observations have been undertaken.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

The Preparation of Glutathione Containing Radioactive Sulfur.—RACHEL G. FRANKLIN. (*Science*, **89**: 298, 1939.) That sulfur-containing compounds are essential to the life and growth of animals has long been known. Such compounds as cystine, an amino acid containing sulfur, or glutathione, a tripeptide, of which cystine is a part, have been found in most of the body tissues; but the exact role which they play in metabolism, or the mechanism of their synthesis has not yet been discovered. It is presumable that if it were possible to label a sulfur compound, fed to an animal, its course throughout the body could then be traced, and in this way information be obtained as to which organs are most concerned in sulfur metabolism, what part these compounds play in the body, how they are formed and why they are so essential to life. The present work was undertaken to find a method of obtaining such a compound.

A means of labelling atoms has been provided in the recently invented cyclotron, which is capable of producing artificially radioactive atoms. Particles can be accelerated in the cyclotron to very great energies and allowed to bombard ordinary elements, resulting in the formation of the radioactive isotopes of the bombarded element, or, in some cases, of new elements. Thus the bombardment of sulfur by 10 Mev. deuterons accelerated by the cyclotron of the Biochemical Research Foundation resulted in the conversion of a part of the sulfur into its radioactive isotope S^{35} according to the following reaction

$$_{16}S^{34} + {}_{1}H^{2} \rightarrow {}_{16}S^{35} + {}_{1}H^{1}.$$

The atom S^{35} cannot be differentiated from ordinary S atoms except by the fact that it emits beta rays, which may be detected by means of a Geiger counter. The radioactive sulfur will enter into chemical reactions which characterize ordinary sulfur; but wherever it is, it is "labelled" by its

radioactivity and can be followed throughout its course. S^{35} has a comparatively long life,—after 88 days only half of it has disintegrated—which permits its use over fairly long periods of time.

The chemical synthesis of sulfur compounds of the type mentioned above presents a difficult problem. It was, therefore, decided to use a biological method. Briefly, this method consisted in growing yeast in a medium containing radioactive sulfur in a form which is easily assimilated by the yeast in its growth, since it is a well known fact that yeast can synthesize glutathione. Subsequently the glutathione was extracted from the yeast thus obtained and found to be radioactive.

The sulfur bombarded in the cyclotron was converted into magnesium sulfate by oxidation in a stream of oxygen in the presence of platinum catalysts at 600° C. The SO_3 vapors formed were led into an absorption chamber containing 5% solution of H_2O_2 and an excess of magnesium oxide, resulting in the formation of magnesium sulfate. The excess of magnesium oxide was filtered out and the solution freed from H_2O_2 by boiling. The radioactive $MgSO_4$ was added to the medium used in the culture of the yeast in a concentration of 0.25 grams per liter.

The strain of yeast used was *Saccharomyces cerevisiae*, Frohberg type, which is known to produce considerable quantities of glutathione. Daily determinations were made on the yeast for its glutathione content. It was found that the yield reached a maximum after three days, which period of time was therefore chosen as a growth period.

After the yeast had grown for three days it was separated from the medium by centrifuging and the glutathione isolated as the highly specific cuprous derivative, by a modification of Pirie's method. Thirty grams of moist yeast containing approximately 30 milligrams of reduced glutathione yielded 20 milligrams of cuprous salt exhibiting the silvery sheen characteristic of this compound in aqueous solution. A portion of the cuprous salt was freed from copper by means of H_2S; the resulting solution was found to be capable of activating the enzyme glyoxalase, thus establishing the presence of free glutathione in the solution. The remainder of the

cuprous salt was repeatedly washed by centrifuging until the washings gave no positive test for sulfate. A sample of this material containing 0.3 mg. sulfur, gave a count of 150 per minute in the Geiger counter. Another portion, containing 0.25 mg. sulfur, was ignited and the sulfate formed precipitated as barium sulfate. This gave a count of 100 per minute. These measurements are only approximately quantitative, due to the self-absorption of the beta rays within the different samples.

The Measurement of Radiations Emitted by Radioactive Compounds.—RACHEL G. FRANKLIN. The success of experiments involving the use of radioactive isotopes as indicators depends to a large extent upon the reliability of the methods used in tracing the "labelled" atoms, or, in other words, upon the accuracy of the activity measurements. Radioactive elements are known to emit three types of rays, alpha rays, beta rays and gamma rays. The majority of the instruments suitable for quantitative measurement depend upon two of the characteristic properties of these rays—their ability to penetrate matter and to form ions in the material through which they pass.

Alpha rays are in general the least penetrating, being absorbed by a fraction of a millimeter thickness of even a light element such as aluminum, and are therefore not particularly adapted to "indicator" work. Beta rays vary considerably in the energy with which they are emitted from radioactive sources and hence in their penetrating power. Soft beta rays, such as are emitted by radioactive sulfur, also are absorbed by thin sheets of aluminum, and other materials, while hard beta rays, such as are emitted by phosphorus, are able to penetrate comparatively thick layers of these materials. Gamma rays are closely related to x-rays. They too differ in their penetrating power, some being absorbed by a few millimeters of lead and others requiring several centimeters of lead for complete screening.

All of these radiations ionize the gas or other material through which they pass, the number of ions produced being proportional to the intensity of the radiation. A quantitative determination of the ionization produced can, therefore, be taken as a measure of the intensity of the radiation.

For strong sources of radiation an ionization chamber may be used. The rays are allowed to pass through the gas, usually air, enclosed by the chamber and the ions formed are swept onto a "collector" by an applied voltage which is high enough to guarantee that all of them are so collected and measured by a sensitive ammeter or electrometer.

Frequently, however, in work involving the use of radioactive indicators, the quantities of material available are not sufficient for accurate measurement by an ionization chamber and a more sensitive instrument, such as a Geiger-Mueller counter, must be employed. This consists essentially of a metal tube, along the axis of which is stretched a fine wire. These are enclosed in a glass tube which can be evacuated to any desired pressure, usually 6–10 cm. of mercury. Between the wire and the metal tube a voltage is applied. Any ionizing ray entering the tube will form ions in the gas in the tube which will be carried to the collecting wire by the potential. A tiny electrical impulse is set up which is amplified by a suitable amplification system. A loud speaker or other device may be used to detect the number of impulses.

Geiger counters are subject to certain inherent errors, for which allowances must be made. Great care must be taken in making and cleaning them, to insure reliability and avoid spurious counts. The voltage must be kept constant. Since the impulses coming to the counter from the radioactive source are not regularly spaced, a large number of counts must be taken in order to get an accurate average. Again, if the impulses arrive in too rapid succession, several impulses may be recorded as only one. Corrections, computed statistically, for these errors may be applied. More recently scaling circuits, which will record only a known fraction of the impulses arriving at the counter, have been used in connection with a counter to reduce errors from these sources. There is always a "background" count due to cosmic radiation or to contamination, which must be subtracted from the apparent count to give the true count of the sample.

Geiger counters can be adapted to meet special requirements. For radiations which are more penetrating, such as gamma rays, ordinary glass tubing may be used for enclosing

the counter, without serious loss by absorption. Phosphorus emits fairly penetrating beta rays, which can be detected by a "thin walled" counter. Considerable work has been done by this method on the role which phosphorus plays in metabolism. Less penetrating radiations, such as the beta rays emitted by radioactive sulfur, are absorbed to a great extent, even by thin sheets of glass, and for their measurement, specially designed counters are necessary. A diagram of a counter in use in the Biochemical Research Foundation is shown in Fig. 1. We are indebted to Mr. A. G. Nester of the

FIG. 1. View *A*

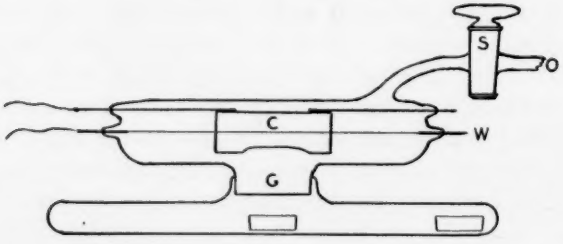


Diagram of Geiger-Mueller counter for the measurement of sulfur radiations.

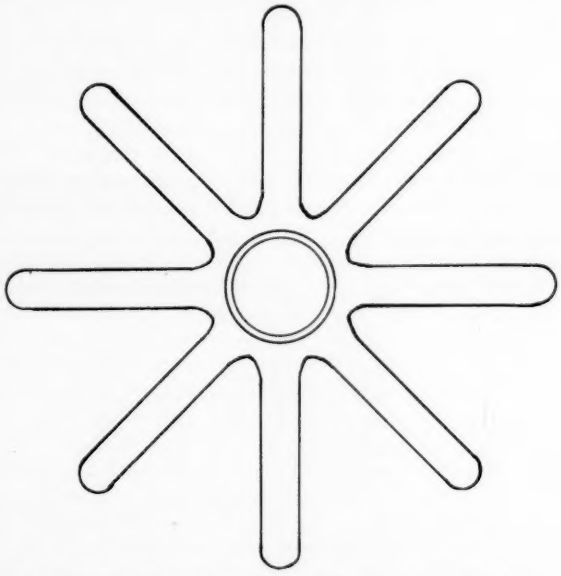
Bartol Research Foundation for help in designing and constructing this apparatus.

This counter consists of two parts, attached by a vacuum tight, ground glass joint, as shown in view *A*. One part contains the counter tube and the other, shown separately in view *B*, the samples to be measured. Copper was used for the metal cylinder *C* in the lower part of which a hole was cut to admit the beta rays. Along the axis of the cylinder a fine tungsten wire is stretched. Between the wire and the cylinder a potential of about 1,300 volts is applied. The sample holder has 8 separate tubes or arms, as shown in view *B*, into which 8 different samples may be introduced. After the sample holder is loaded and attached, the tube is evacuated through the outlet *O* to a pressure of 8 cm. of mercury, and the stopcock *S* closed. The samples, which are in small nickel boats, can be moved one at a time by a magnet into place under the counter for measurement. The rays from those in the side arms cannot penetrate the glass and so cannot

affect the counting rate. In this way several samples can be measured under identical conditions of pressure and voltage and accurate comparisons can be made with a suitable standard. The counter may be cleaned easily in case of contamination.

Care must be exercised in the preparation of the samples to insure their similarity if comparative results are to be expected. The beta rays are so soft that if the samples are too

FIG. 1. View *B*.



The sample holder of the Geiger-Mueller counter as seen from below.

thick those rays coming from the lower layers are absorbed before they reach the surface. It has been found possible to obtain good results by converting the sulfur compounds to be measured into barium sulfate. Equal parts are weighed onto the small nickel boats and suspended evenly in a few drops of water. The water is allowed to evaporate slowly, leaving a thin uniform layer. A drop of a 0.01% solution of agar added to the suspension before evaporation prevents accidental loss during measurement. A counter of this type can detect very minute quantities of radioactive sulfur.

BOOK REVIEWS.

THE GEOMETRY OF DETERMINANTAL LOCI, by T. G. Room. 483 pages, illustrations, 19 × 28 cms. New York, The Macmillan Company, 1938. Price $10.

The subject matter of this book holds fascination for many mathematicians, particularly those interested in research in geometry. There is still much unexplored territory and the problems contained therein offer a means for exercise and ingenuity. The author of this book states that it is the result of 15 years work. It should provide a fair start along lines of this branch of mathematical investigation.

The plan of the book is such that general theorems are first established, followed by a discussion of the particular configurations which appear to be of greatest interest. The point made by this method is the saving in time and labor, although the investigation of the peculiarities of special loci and configurations are of prime importance. After an introductory chapter which sets out the suffix and summation conventions adopted in the book and discusses briefly the meanings of terms in current use in projective geometry, the book is divided into three parts with a total of 25 chapters.

Part I has the general title of manifolds defined by matrices of general form. Under this a survey is made of the fundamental properties of manifolds, and there are discussed three different ways of establishing a birational correspondence between two projectively generated manifolds, the application of the degeneration principle, the pairing theorems, the key manifolds, determinantal primals, and the freedom of a projectively generated manifold. This material is more or less of a general nature and may be passed over by some and used only for reference.

The second part of the book has to do with manifolds defined by matrices of special forms. In it there is attention given to key-manifolds, manifolds representable by the vanishing of minors of a symmetrical determinant, those defined by skew symmetrical determinants, normal rational scrolls, and representations of spaces by points. Finally there are given some generalizations of the classic theorem of the "fifth associated line" containing a generalization of the Segre primal.

Determinantal quartic primals in space of four dimensions is the heading of part III of the book. This covers almost 100 pages and is followed by appendixes on Veroneseans and Grassmannians, the characteristic numbers of certain determinantal manifolds, and the freedoms of manifolds. An index is in the back.

The book provides a course of study that should prove illuminating as well as inspirational to those qualified for its perusal.

R. H. OPPERMANN.

SIR CHANDRASEKHARA VENKATA RAMAN, 1888—7th November—1938. Jubilee volume of original papers presented in commemoration of his fiftieth birthday. 322 pages, illustrations, plates, 20 × 27 cms. Bangalore, Indian Academy of Sciences, 1938.

Every scientist is familiar with at least some of the works of Sir C. V. Raman, the best known of which is his discovery in 1928 of the radiation effect known by

his name. Since that time a great volume of literature has grown up around the subject that has thrown new light on many problems of Physics and Chemistry. But Sir Raman has had a career that has placed him among the foremost in many lines of endeavor. He is recognized as being the leader in the promotion of scientific thinking and research in India. As a result, India today is making valuable contributions not only in pure physics but also in fields such as chemistry, meteorology, and seismology.

The Indian Academy of Sciences has taken the fiftieth birthday of Sir Raman as an occasion to signalize the completion of ten years international collaboration and research on the Raman effect. Accordingly, in inviting contributions to this volume, they indicated the scattering of light, the Raman effect and related topics as the preferred subject. In response there are here recorded some forty contributions from authors who have so honored Sir Raman. Each is a distinguished man in his field. A perusal of these papers will reveal much of value and inspiration.

R. H. OPPERMANN.

GENERAL AND INORGANIC CHEMISTRY, by Frederick C. Irwin and G. Ray Sherwood. 583 pages, plates, illustrations, 16 × 22 cms. Philadelphia, P. Blakiston's Son & Co., Inc. Price $3.50.

With the rapid growth of modern chemistry, it has been recognized by those whose responsibility it is to teach that science, that progress of students in its study must be accelerated at no expense to understanding or completeness of coverage. As a result many experiments have been made in the technique of presentation, and some of the results have been generally adopted, other dismissed. In this respect various constructions in books have been arranged, some good and some not so good, but all with the purpose of fitting into a special scheme. On the one hand there is the text of a purely descriptive or "short cut" nature intended merely to touch the high points sufficient to create enough genuine interest for the pursuit of the subject in greater detail, and on the other the old fashioned text which is colorless and tiresome.

This book may be placed somewhere between these two extremes. Various devices are used to maintain interest, pitfalls are pointed out, and daily observations are cited. At the same time actual exercises are injected, so that a comprehensive treatment of fundamentals is provided. The introduction to the work is through a brief discussion on some properties of matter in order to refresh those who have studied this and to provide a basis for those who have not. This is followed by the kinetic theory, chemical reactions, molecular theory and weights, classification and discussion of the elements and various chemical compounds. Of particular interest here is the attempt to show why the periodic law is true, why atoms do or do not combine, and why compounds have certain physical or chemical properties.

The study of oxygen compounds of the halogens then appears, succeeded by a development in the field of solutions beginning with the definition of units of measurement and proceeding through Raoult's law of vapor pressure, a discussion of freezing point depression and boiling point elevation, the laws of osmotic pressure, and a brief outline of the bombardment theory. Subsequently there are covered electrolytes, sulphur, oxides and oxygen acids of sulphur, nitrogen and

ammonia, and acids. At this point a chapter appears on density, units of force and energy relations which reviews evidence of atomic structure. This will be helpful to those students who may for some time have been out of touch with these fundamentals of physics—a typical device used by the author designed in the direction of helpfulness.

The spectroscope and spectra are presented via reference to helium and the rare gasses, and X-ray spectra and crystal structure in like manner. The study of the metals is taken up in similar plan, beginning with the most active chemically, the alkali-metals (mainly aluminum), copper, silver, lead, gold, and mercury, completing with iron and steel, chromium and manganese. The concluding chapter of the book is on colloids.

The book follows a logical sequence, although there are variations from the usual. The main feature of the book is clarity. Any serious minded student with somewhat less than the usual preparation should be able to follow the presentation with little outside assistance.

R. H. OPPERMANN.

SAMPLING AND ANALYSIS OF CARBON AND ALLOY STEELS. Methods of the chemists of the subsidiary companies of the United States Steel Corporation as revised to 1937. New York, Reinhold Publishing Corporation, 1938. Price $4.50.

As those who are familiar with the two former booklets dealing with this subject are aware, the chemists of the United States Steel Corp. have been giving coöperative effort in developing, investigating, and standardizing methods. This book replaces those two former booklets, revising older methods and incorporating newer ones. The subjects covered include sampling, methods of analysis, chemical separations, determination of manganese, phosphorus, sulphur, silicon, copper, nickel, chromium, vanadium, molybdenum, titanium, aluminum, arsenic, tin, cobalt, tungsten, uranium, zirconium, columbium and tantalum, selenium, boron, beryllium, nitrogen, antimony, copper and molydenum, zinc, and cerium. Appendixes give determination of sulphur or carbon and sulphur by combustion in oxygen; discussions of methods for oxygen or oxygen, hydrogen and nitrogen in steel; preparation, pH ranges, and color changes of various organic chemical indicators. It is complete with all data necessary and will be a valuable addition as a working tool for all those whose interests lie in this direction.

R. H. OPPERMANN.

PHYSICO-CHEMICAL EXPERIMENTS, by Robert Livingston. 257 pages, illustrations, 15 × 22 cms. New York, The Macmillan Company, 1939. Price $2.25.

The performance of successful instruction involves a knowledge of what details to omit as well as a knowledge of how far and how complete certain directions should be given. To encourage initiative, develop a sense of responsibility, and emphasize the flexibility of procedures and knowledge, are aims which require methods of extreme care. The modern laboratory course must have these in the background and accomplish them at no expense of minimizing in any way interest in the subject.

Physico-Chemical Experiments is a laboratory manual which is an illustra-

tion of present day concepts of these aims. It covers the fundamental principles of the subject, giving practical experimentation and computation necessary to correlate the abstract principles and equations of theoretical chemistry with the experimental observations upon which they are based. In doing this, familiarity is made with the technique of the common laboratory apparatus when performing experiments.

The book opens with quite an extensive treatment on measurements, errors and computations. This outlines the basic theory of physical measurements and laws, and numerical and graphic methods of mathematical approximation which are most commonly applied in the analysis of physical measurements. After this there are descriptions of 41 experiments on 17 different subjects. The list is headed with the subject of the gaseous state and experiments on the determination of the molecular weight of a vapor by the Dumas method, by the Victor Meyer method, and on the effusion of gases are described under it. A description of the barometer and its use are also given. Under other subject headings such as the liquid, solid, and colloidal states, the structure of atoms, equilibrium, electrical conductance, and electrolysis pertinent experiments are listed likewise.

The method of describing the experiments is noteworthy. It begins with a brief discussion of the subject generally which inclines toward the reasons for the experiment, what it is expected to produce, and hints on difficulties to be expected in its performance. Sometimes certain apparatus is described as in the case of the above mentioned barometer or in the case of the potentiometer. The apparatus necessary for the experiment are named, followed by the procedure, and finally the observations and certain computations necessary are pointed out. The experiments are of varying length and difficulty and the directions are sufficient for performance without assistance. There are no directions for preparing reports, the assumption being that the student should take this responsibility.

The selection of the experiments and their exposition is exceptionally well done. The manual can be used with any good modern text on physical chemistry and the use of the comprehensive subject index in the back should make the book of value as a reference.

R. H. OPPERMANN.

ASSOCIATION OF CONSULTING CHEMISTS AND CHEMICAL ENGINEERS, INC. Directory of Association Members, January 1, 1939. Fifth Edition, 16 pages, 21 × 28 cms. New York, Association.

This useful booklet is designed to assist all who need chemical advice or service. It is a directory of members of the Association, listing them alphabetically and geographically. There are one page statements from each member, descriptive of his and his organization's qualifications, scope, functions and activities. At the end there is a key sheet of various types of work handled by the members under two qualifications—specially qualified by exceptional training and experience, and generally qualified. Anyone in need of the services of a chemist or chemical engineer in a consulting capacity would do well to examine this booklet. It may be obtained gratis on applying to the office of the Association at 50 E. 41 St., N. Y. C.

R. H. OPPERMANN.

DIRECT-CURRENT MACHINERY, by Hempstead S. Bull. 318 pages, illustrations, 15 × 24 cms. New York, John Wiley & Sons, Inc., 1939. Price $3.00.

A very important part of electric power engineering is that on direct current machinery. It is the tangible accomplishment of the application of electrical theories and laws, knowledge of which was previously acquired in the study of electricity and magnetism, and it paves the way for more advanced study of direct currents and alternating currents. This position of the subject of direct currents in the study of electrical engineering is a fortunate one for students because it has immediate and practical application early in the course, giving a tremendous impetus to further study. This fact, together with the proper weight which direct current machinery must be given in the practice of electrical engineering, makes it imperative that a text on the subject be very carefully drawn.

The book at hand is designed to meet these requirements. To use it to advantage requires a thorough grounding in elementary physics with special regard to electricity and magnetism, a knowledge of trigonometry, differential calculus, with, perhaps, a familiarity with analytical geometry. These prerequisites constitute no more than any beginner should have for the study of the engineering of direct current machinery. The author wastes little space in introducing his subject by way of review of these, giving only sufficient material to refresh memory and to place the student in position for the sequence which follows. The introduction features the force acting upon a conductor in a magnetic field, which is used as the starting point of the text proper, being employed in the next chapter on measuring instruments. From here motor elements are taken up with explanations of the commutator, armature coils, and the structure of the motor—a descriptive chapter. The technical study is resumed with the shunt motor which leads to treatments on the series motor by indicating the effect of changing the shunt voltage and the introduction of a series winding in the compound motor. This section is terminated with coverage on starters and speed control.

Generators are treated by way of first studying the magnetic circuit, induction, and then generator characteristics and performance. More detailed information is given on the various methods of interconnecting the armature conductors so that the desired force and voltage effect are produced efficiently and economically. There is given a view of motors and generators as transformers of power, which covers losses and efficiency. Further necessary details such as armature reaction, commutation, parallel operation of generators, and the operating characteristics of boosters, balancers and other special generators are considered. The last chapter of the book is devoted to electric circuits which analyzes the practical phase of circuits and networks. There is useful appendix material and a set of tables and curves giving data of a helpful nature. A subject index completes the book.

The arrangement of the subject matter as indicated above is somewhat unusual. It reveals a keen insight of the situation and needs of the student which results in a modern and outstanding presentation.

R. H. OPPERMANN.

POWER-FACTOR ECONOMICS, by Price L. Rogers. 143 pages, illustrations, tables, 15 × 24 cms. New York, John Wiley & Sons, Inc., 1939. Price $2.50.

Any engineer who has had opportunity to observe the beneficial results in the way of better performance, operating efficiency, and economy of electric power machinery due to the proper application of electricity supply, can appreciate the value of power factor economics. The subject is a specialized part of electric power engineering by reason of the fact that to become proficient in it requires, in addition to a thorough knowledge of theory, considerable experience with a wide variety of conditions. The trend of the literature on power factor correction over the past 15 years runs parallel with other literature on applied engineering—toward the lessening of the amount of experience required for practical application. The book at hand represents the modern state of the art in the literature.

It reviews briefly the theoretical elements which affect power factor, the operating characteristics of rotating and stationary machinery, the effects produced by power factor, and corrective apparatus in description and size necessary to obtain favorable results. The major portion of the book is devoted to the industrial phases of the subject which is completed with a number of worked out studies of correction in industrial plants. In this there is illustrated application to purchased power situations with the most common utility rate structures.

The other part of the book is devoted to power factor on utilities distribution systems. Certain types of correction here are now really coming into general practice, making considerable savings in line and equipment capacity, as well as removing undesirable conditions due to the use of small motor driven appliances in households. The use of different types of capacitors, with formulas and other data, together with the procedure for the solution of operating problems is given.

Power factor economics remains a specialized part of electric power engineering, depending upon practical experience gained in the solution of actual problems. The book, however, does give a lot of up-to-the-minute useful data. It provides a basis for the electrical engineer who contemplates entering this field and directs him into the right channels for further work. The experienced specialist too may find useful information.

R. H. OPPERMANN.

PHYSICAL METEOROLOGY, by John G. Albright. 392 pages, plates, illustrations, 16 × 24 cms. New York, Prentice-Hall, Inc., 1939. Price $5.35.

The weather is one of those things which unknowingly is playing an increasingly important part in everyday affairs of all classes of people. In agriculture and at sea, weather has long been recognized as a vital factor, but with the present day railroads, automobiles, electric transmission lines, airplanes, and a host of other common place utilities the weather plays its part. It is true that the vagaries of the weather still exist and nothing can be done about them. It is also true that the progress made in knowledge of the causes of different kinds of weather enable more accurate forecasts which result in more enjoyable and productive life.

This book is a text on atmospherics and climatology. It is an elementary treatment on the various physical aspects of the subject and the operation of physical principles and laws producing the seemingly unsystematic perturbations

of the atmosphere, especially as applied to methods of anticipating weather conditions. In order to get the maximum from its use, the usual prerequisites for a college course are required including physics, and mathematics, although the book is so written that it may be used profitably by those not having much more than a smattering of these. The treatment begins with the study of the composition of the air, its physical properties, density, elasticity and specific heat, and its absorption power for radiation passing through it. Wind directions, the dynamics of air movements, condensation of atmospheric moisture, cyclones, atmospheric electricity, storms, and atmospheric acoustics and optics are topics which follow.

The treatment is about equal in explanatory matter and in problem work. The former is presented in the manner that those not having all of the required background can read understandingly. At the end of each chapter there is a set of questions to cover this phase. A list of problems appears at the end of chapters in which such material is given.

The subject is one which is fascinating to everyone with an inquiring mind. In addition to this there is the economic value of the subject with its growing demand for forecasts on the ground and for elevations far above it. The coverage given in this book is of sufficient extent and of an informative and inspirational character to satisfy both, providing cultural as well as practical knowledge.

R. H. OPPERMANN.

A HISTORY OF SCIENCE, TECHNOLOGY, AND PHILOSOPHY IN THE EIGHTEENTH CENTURY, by A. Wolf. 814 pages, illustrations, plates, 16 × 25 cms. New York, The Macmillan Company, 1939. Price $8.00.

When a book of this nature comes to hand one cannot but help reflect on present day affairs with respect to many of the subjects covered. A sage may observe on the youthful aspirations and beliefs of each new generation—they believe they are the chosen few of all, they are wiser, they are more creative, they can surmount greater difficulties. The apparent gullibility of many for so-called new ideas for the solution of complex problems, particularly on social sciences and economics is an example. There seems to be a lack of knowledge of the work of those who have gone before or a lack of appreciation of their work. In technology too, this is often the case in spite of the fact that it may concern the application of laws known by the name of their discoverer such as Boyle or Maxwell or Charles. There is undoubtedly a great practical as well as cultural value to be obtained from a book on history of science, technology, and philosophy.

This book covers these subjects for the eighteenth century, that period which has given so much toward a basis for our present progress. It gives a comprehensive knowledge of achievements which penetrate to a considerable extent into detailed descriptions. Divided into the three parts of its title it tells in the form of a smooth running account how mathematics was extended and systematized, of new generalizations in mechanics, the construction of the vast dynamic system in astronomy on a Newtonian basis; the progress in physics and its branches, the advance in meteorology, chemistry, geology, geographical exploration, and the biological sciences. With regard to technology, the book tells of improvements in agricultural methods and implements, inventions in the textile industry, building construction and other construction, the steam engine, balloons and para-

chutes, machine tools, gas illumination, and the start made in industrial chemistry with the large scale manufacture of sulphuric acid and alkali.

Under the heading of Philosophical Studies there is found psychology, the social sciences and philosophy. Of these, psychology was most popular in the eighteenth century. It is shown as being applied to education and the formation of national character. Statistics, economics and philosophy likewise profited during this period.

Readers of many interests will find here much of value. The index is very helpful from a reference standpoint. As an illustration, with the use of it there can be found the extent to which reference is made to the work of Benjamin Franklin, of his theories, beliefs, and accomplishments in electricity and magnetism and of his Pennsylvanian Fire-place. The same can be done with Franklin's associates and friends in scientific endeavor. Another work referred to of particular interest at this time is that of Adam Smith. The book by its very nature must limit the amount of detailed accomplishments, even approaching the point of ignoring some.

The book represents a tremendous effort at complete coverage for this period. It is a monumental work that gives a definite focus on the pertinent subjects with a blending of bordering related material. It should be on the shelves of everyone who takes pride in the collection of good reference literature.

R. H. OPPERMANN.

PUBLICATIONS RECEIVED.

Physical Constants of Hydrocarbons, by Gustav Egloff. Vol. 1, Paraffins, Olefins, Acetylenes, and other Aliphatic Hydrocarbons. 403 pages, 16 × 24 cms. New York, Reinhold Publishing Corporation, 1939. Price $9.00.

Modern Magnetism, by L. F. Bates. 340 pages, illustrations, 15 × 22 cms. New York, Cambridge University Press, 1939. Price $4.50.

Protective Coatings for Metals, by R. M. Burns and A. E. Schuh. 407 pages, illustrations, 16 × 24 cms. New York, Reinhold Publishing Corporation, 1939. Price $6.50.

An Introduction to Crystal Chemistry, by R. C. Evans. 388 pages, illustrations, 15 × 22 cms. New York, Cambridge University Press, 1939.

Electron Optics, by Otto Klemperer. 107 pages, illustrations, 14 × 22 cms. New York, The Macmillan Company, 1939. Price $1.75.

Le Soja et les Industries du Soja, by A. Matagrin. 390 pages, illustrations, 14 × 18 cms. Paris, Gauthier-Villars, 1939. Price 60 francs.

High-Frequency Alternating Currents, by Knox McIlwain and J. G. Brainerd. Second Edition, 530 pages, illustrations, 16 × 24 cms. New York, John Wiley & Sons, Inc., 1939. Price $6.00.

Casein and Its Industrial Applications, by Edwin Sutermeister and Frederick L. Browne. Second Edition, 433 pages, illustrations, plates, 16 × 24 cms. New York, Reinhold Publishing Corporation, 1939. Price $6.50.

Steam Conquers the Atlantic, by David Budlong Tyler. 425 pages, illustrations, plates, 16 × 24 cms. New York, D. Appleton-Century Company, 1939. Price $5.00.

Association of Consulting Chemists and Chemical Engineers, Inc. Directory of Association Members, January 1, 1939. Fifth Edition, 16 pages, 21 × 28 cms. New York, Association.

A Preview of the Absolute-Relative Theory, by Franklin J. Matchette. 26 pages, illustrations, 18 × 26 cms. New York, Author, 1939. Price 50 cents.

A Mathematical Prodigy: History and Legend, by Cassius Jackson Keyser. 12 pages, plates, 17 × 25 cms. New York, Scripta Mathematica, 1938. Price 20 cents.

Greek Mathematics and Astronomy, by Sir Thomas Little Heath. 18 pages, 17 × 25 cms. New York, Scripta Mathematica, 1938. Price 25 cents.

National Advisory Committee for Aeronautics. Technical Notes: No. 690, Resume of Air-Load Data on Slats and Flaps, by Carl J. Wenzinger and Francis M. Rogallo. 36 pages, illustrations. No. 691, Some Elementary Principles of Shell Stress Analysis with Notes on the Use of the Shear Center, by Paul Kuhn. 45 pages, illustrations. No. 692, Some Fundamental Considerations in Regard to the Use of Power in Landing an Airplane, by Walter S. Diehl. 9 pages, illustrations. No. 693, Comparison of Profile-Drag and Boundary-Layer Measurements Obtained in Flight and in the Full-Scale Wind Tunnel, by Harry J. Goett and Joseph Bicknell. 15 pages, illustrations. No. 694, Physical Properties of Synthetic Resin Materials, by Meyer Fishbein. 32 pages, illustrations. No. 695, The Effects of Some Common Surface Irregularities on Wing Drag, by Manley J. Hood. 33 pages, illustrations. No. 696, Torsional Stability of Aluminum Alloy Seamless Tubing, by R. L. Moore and D. A. Paul. 20 pages, illustrations. No. 697, The Frequency of Torsional Vibration of a Tapered Beam, by Robert P. Coleman. 21 pages, illustrations. No. 698, Propeller Tests to Determine the Effect of Number of Blades at Two Typical Solidities, by E. P. Lesley. 20 pages, illustrations. 9 pamphlets, 20 × 26 cms. Washington, Committee, 1939.

Bell Telephone System, Monographs: B-1101, Ultra-Short-Wave Transmission and Atmospheric Irregularities, by C. R. Englund, A. B. Crawford and W. W. Mumford. 30 pages, illustrations. B-1113, A Short-Wave Single-Side-Band Radiotelephone System, by A. A. Oswald. 24 pages, illustrations. B-1114, A Single-Side-Band Receiver for Short-Wave Telephone Service, by A. A. Roetken. 11 pages, illustrations. B-1117, The Oxide-Coated Filament, by C. H. Prescott, Jr., and James Morrison. 15 pages, illustrations. 4 pamphlets, 15 × 23 cms. New York, Bell Laboratories, 1938.

CURRENT TOPICS.

New Aluminum Process Claimed.—(*Industrial and Engineering Chemistry News Edition*, Vol. 16, No. 21.) HERBERT LEOPOLD of Tokyo, Japan, reports that following three years of experimentation, the Dai Nippon Sugar Refining Co. has decided to undertake the manufacture of aluminum by a process credited to Kato and Funaki of Tokyo University. According to available details, the new plant will use a phosphorus-bearing alumina which has been discovered in Manchuria. The ore is treated with sulfuric acid and the alumina, aluminum sulfate. The latter is treated with gaseous ammonia at 1200 deg. C., the products being ammonium sulfate and aluminum oxide. Reduction of the alumina is by the usual electrolytic method. It is planned to treat 15,000 tons of ore annually, producing therefrom 5100 tons metallic aluminum, 25,800 tons of ammonium sulfate, and 20,000 tons of fertilizer containing 20 per cent. phosphoric acid and 16 per cent. ammonium sulfate. The plant is under construction at Hachinohe, where there is a favorable supply of power. No information is available regarding the cost of this process in comparison with the well known practice of producing aluminum from bauxite. The new concern believes itself in a favorable position, particularly since it is anticipated that the task of obtaining adequate supplies of bauxite promises to become more and more difficult.

R. H. O.

Potato Starch.—Equipped with newly designed time saving machinery for the manufacture of starch for New England Textile mills, two potato starch plants are nearing completion at Houlton, Maine. The two mills, using surplus potato crops, are reported to mark a notable advance in the process of American starch manufacturing and point to a wider use of agricultural products in industrial fields. The two plants are being constructed by the New England Starch Company and the Aroostook Potato Products, Inc., the latter being a subsidiary of Morningstar, Nicol, Inc. The New England Starch Company will use approximately 2,000 barrels of potatoes a day and have a maximum production capacity of 20 tons of finished starch a day. In this mill the "time cycle" of manufacturing will be reduced from the usual 72 hours to six hours. At the Aroostook Potato Products plant a new type of American

starch will be manufactured which will compete with the finest of imported starches. In addition to utilizing surplus local crops, the plant also will manufacture from imported raw materials.

R. H. O.

Government Explores Public Phosphate Lands in Florida.—Public lands in Florida were first withdrawn from entry by President Taft in 1910 as a conservation measure, because of their possible phosphate content. The area reserved was subsequently modified by further withdrawals and restorations so that on January 1, 1934, it contained 66,796 acres, but only a small part of this area had been definitely classified as phosphate land. In 1934 and 1935, under two successive grants from the Public Works Administration, the Geological Survey examined 83 tracts of these lands in Polk County, comprising in all about 3,300 acres, and 6 tracts in Citrus and Marion Counties, comprising 240 acres. The lands in Polk County were all in the so-called land-pebble field, and the others in the hard-rock field. The object of the investigation was to obtain data on which to base a decision whether the phosphate rock present in any given tract is sufficient in quantity or of suitable quality to justify the Government in retaining the mineral rights in that tract. In phosphate mining in the Florida field attention at the present time is concentrated on material that contains 70 per cent. or more of tricalcium phosphate ($Ca_3(PO_4)_2$), called in the trade "B.P.L." (bone phosphate of lime), but that contains less than 3 per cent. of iron and alumina (abbreviated to I and A). High-grade rock of this kind has been much in demand for the manufacture of superphosphates by the acidulation process. However, operations of the Tennessee Valley Authority and others have shown that by the use of the electric furnace material a much lower grade can be successfully utilized, and it is thought that by this and other possible methods material even as low as 40 per cent. in B.P.L. content may eventually be used. The results are grouped according to the grade of the phosphate pebble recovered. The results listed are all for the pebble field in Polk County. None of the wells drilled in the hard-rock field yielded material of grade as high as 40 per cent. B.P.L., but as the distribution of phosphate in the hard-rock field is known to be very irregular the results obtained would probably justify in most of the tested areas retention of mineral rights by the Government.

R. H. O.